Flagellar locomotion of micro-organisms

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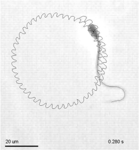
Introduction

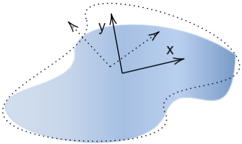
Eutreptiella's "Metaboly" movement.



Bull sperm.

Stokes equation

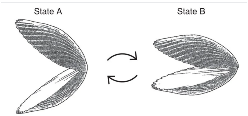
$$-\mu \Delta u + \nabla p = 0, \; div\, u = 0$$

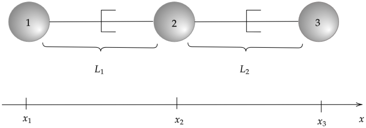


Deformable solid surrounded by a fluid.

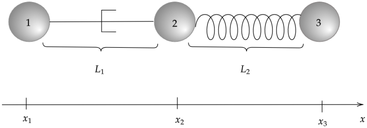
Purcell's Scallop Theorem

- Low Reynolds number $Re = VL/\nu$
- A reciprocal movement cannot lead to a global displacement.





Simple 3-sphere swimmer [1].



3-sphere swimmer with a spring [2].

[1] A. Najafi and R. Golestanian.
[2] A. Montino and A. DeSimone.

N-spring swimmer

Approximations

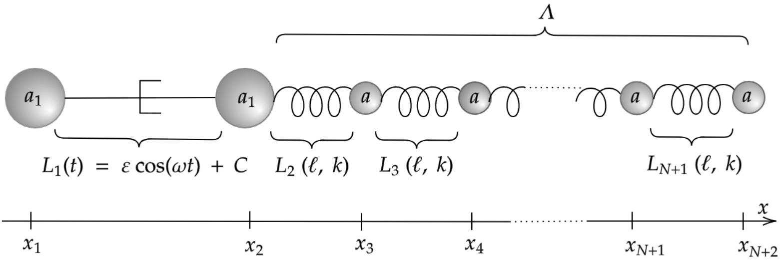
- Linear forces w.r.t. speed:

$$\begin{cases} f_j^F = 6\pi\mu a V_j \text{ pour } j \geq 3\,, \\ f_j^F = 6\pi\mu a_1 V_j \text{ pour } j = 1, 2. \end{cases}$$

- No interaction between spheres.

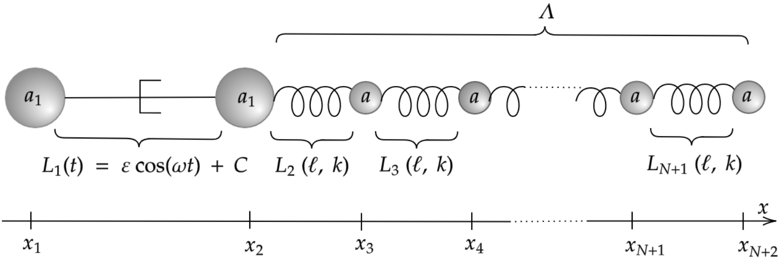
Parameters

- $\ell = \Lambda/N$
- $a = \tilde{a}/N$
- $k = N\tilde{k}$



Equations of motion

$$\begin{cases} \dot{\ell}_j = \frac{K}{\ell^2}(\ell_{j-1} - 2\ell_j + \ell_{j+1})\,, & 2 \le j \le N+1 \\ \dot{\ell}_2 = -\frac{1}{2}N\dot{L}_1 + \frac{N^2\tilde{k}}{6\pi\mu\tilde{a}}(\ell_3 - \ell_2) + \frac{N\tilde{k}}{12\pi\mu a_1}\ell_2\,, & \\ \ell_{N+2} = 0. & \left(K = \frac{\tilde{k}\Lambda^2}{6\pi\mu\tilde{a}}\right) \end{cases}$$

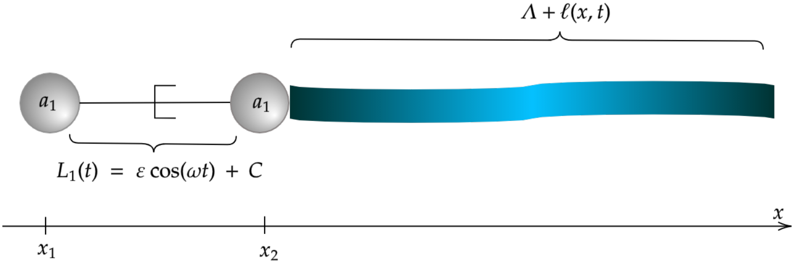


Elastic swimmer

Continuous analogy

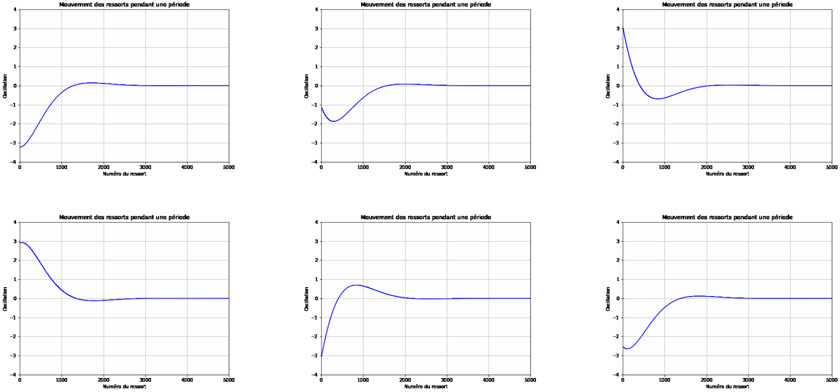
Find $\ell \in \mathcal{C}^2([0,\Lambda] \times \mathbb{R}_+^\star)$, $\forall (x,t) \in]0,\Lambda[\times \mathbb{R}_+^\star$

$$\begin{cases} \frac{\partial \ell}{\partial t}(x,t) = \frac{\tilde{k}\Lambda^2}{6\pi\mu\tilde{a}}\frac{\partial^2 \ell}{\partial x^2}(x,t)\,, \\ \frac{\partial \ell}{\partial x}(0,t) + \frac{\tilde{a}}{2a_1\Lambda}\ell(0,t) = \frac{3\pi\mu\tilde{a}}{\tilde{k}\Lambda}\dot{L}_1(t)\,, \\ \ell(\Lambda,t) = 0. \end{cases}$$



Swimming motion

A wave propagates along the swimmer



Motion of a 5000-spring swimmer for a single stroke.

Machin's swimming rod (1958)

Wave propagation along a flagellum

$$\frac{\partial^4 y}{\partial x^4} = \frac{4\pi\mu}{QSK^2(2.0-log(R))}\frac{\partial y}{\partial t}$$

Four modes:

$$y = e^{i2\pi ft}\left(Ae^{r_1x/l_0} + Be^{r_2x/l_0} + Ce^{r_3x/l_0} + De^{r_4x/l_0}\right)$$

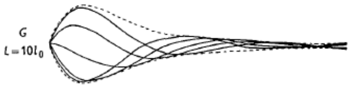
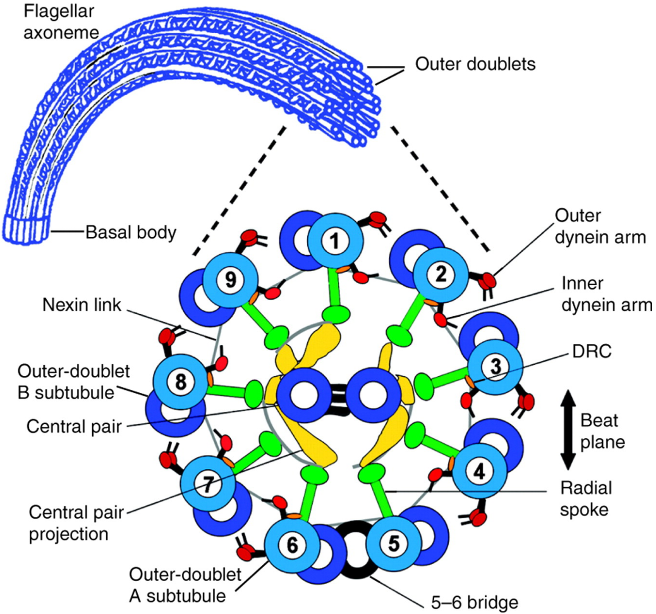


Figure 1: Calculated wave-patterns on a flagellum.

Parameters

- R: Reynold's number
- Q: Young's modulus
- S : area of cross-section
- K: radius of gyration of the section
- l_0: scale length (depending on K, Q, S, R)

Flagellar activation mechanisms



Flagellar activation mechanisms

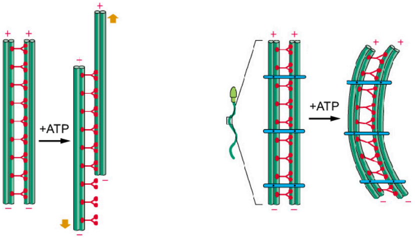


Figure 2: Effect of a spring (nexin links) between two microtubules.

Two-state motor model

$$\begin{cases} M_1 + ATP \rightleftharpoons M_2 + ADP + P \\ M_1 + ADP + P \rightleftharpoons M_2 + ATP \end{cases}$$

$$\omega_1(x) = \alpha(\bar{x})e^{(W_1+\Delta\mu)/k_BT} + \bar{\alpha}(x+\ell/2)e^{W_1/k_BT}$$
$$\omega_2(x) = [\alpha(\bar{x}) + \bar{\alpha}(x+\ell/2)e^{(\Delta\mu)/k_BT}]e^{W_2/k_BT}$$

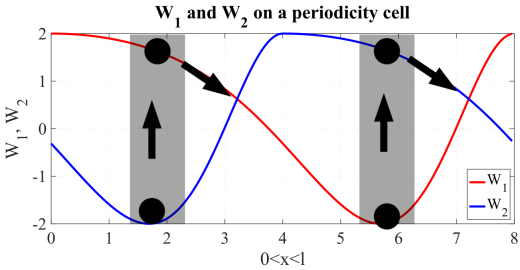
$\Delta\mu \equiv \mu_{ATP} - \mu_{ADP} - \mu_P$.

Mathematical model

Two-state motor model [3] [4]

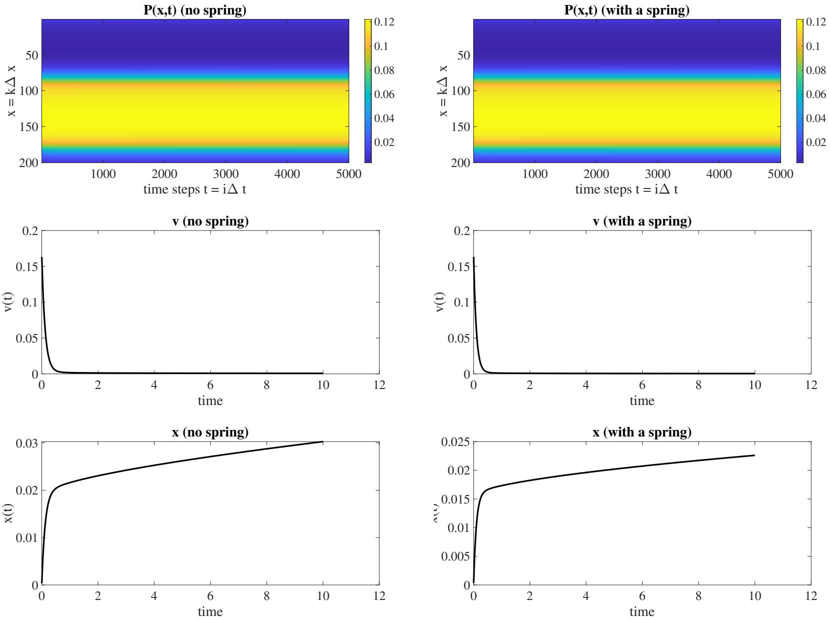
$P = P_1$ probability of being in state 1. $P_2 = 1/\ell - P_1$, $\int_0^\ell (P_1 + P_2) = 1$.

$$
\begin{cases}
\partial_t P + v\partial_x P = -(\omega_1 + \omega_2)P + \omega_2/\ell \\
v = \frac{1}{\eta}\int_0^\ell dx P \partial_x \Delta W - kx + f_{ext}
\end{cases}
$$

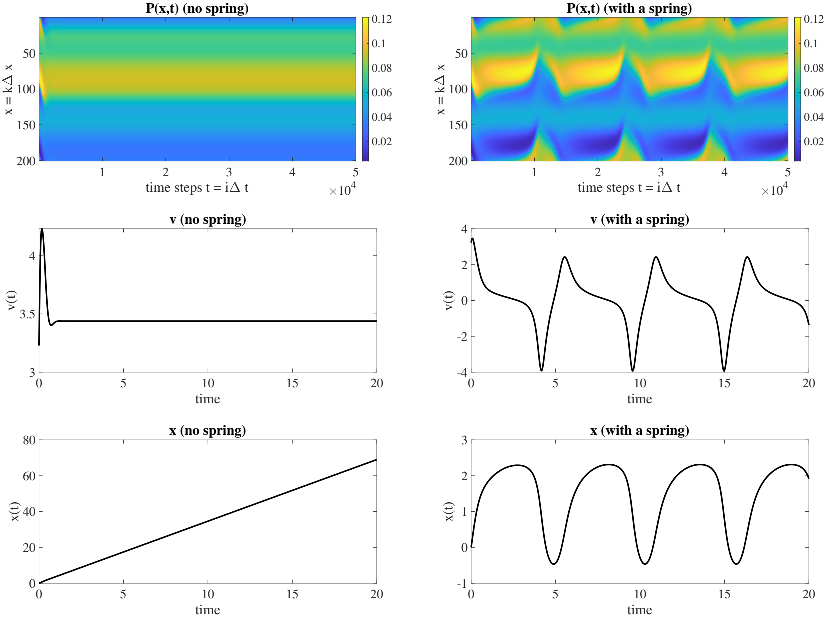


[3] F. Jülicher, ”Lecture notes in physics”, 2007.
[4] F. Jülicher, J. Prost, *Phys. Rev. Lett.*, 1997.

Numerical results ($\Delta\mu = 0$)



Numerical results ($\Delta\mu = 2max(W_1)$)



Conclusion

Final observations

- Activation needed all along flagella;
- Hopf bifurcation induced by ATP concentration creates torsion.

Future prospects

- Model whole axoneme using two alternatively activated sides;
- Use model in a comprehensive sperm-cell simulation.

Thank you for your attention