Wave Propagation in presence of a Periodic Halfspace

Case where the slope of the interface is arbitrary

Pierre Amenoagbadji Sonia Fliss Patrick Joly

POEMS — UMR 7231 CNRS - INRIA - ENSTA Paris - IPP

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Outline

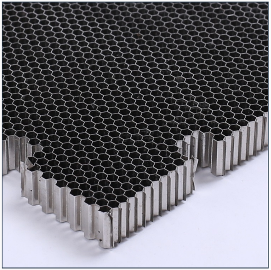
Motivation

Overall objective

Mathematical and numerical analysis and **simulation** of some **wave propagation phenomena** arising in **acoustics**, **elastodynamics** and **electromagnetism** in presence of **heterogeneous media**.

Periodic media

Media whose **geometry** or **physical properties** can be represented as **periodic functions**



Composite materials in mechanics

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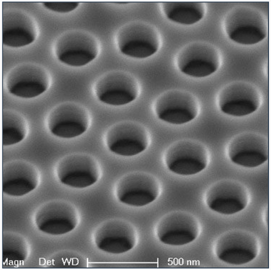
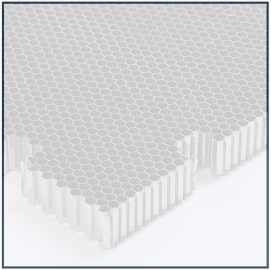


Photonic crystals in optics



Composite materials in mechanics

Typical model

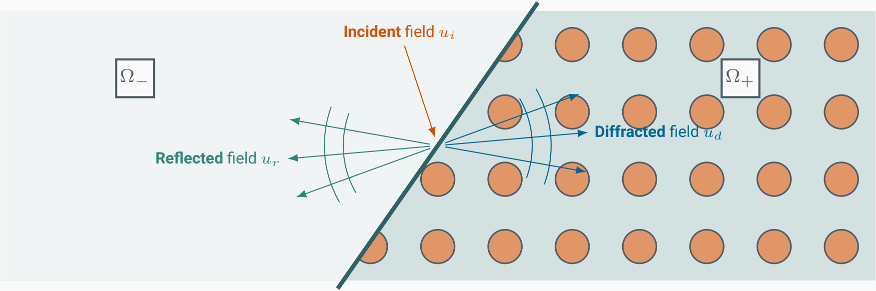
Time-harmonic wave equation

The total field u_t satisfies the **Helmholtz** equation:

$$\operatorname{div} \mathbf{A}_t \nabla u_t - \rho_t\, \omega^2\, u_t = 0, \quad \text{in} \quad \mathbb{R}^2$$

where

$$u_t = \begin{cases} u_i + u_r & \text{in} \quad \Omega_- \\ u_d & \text{in} \quad \Omega_+ \end{cases}$$



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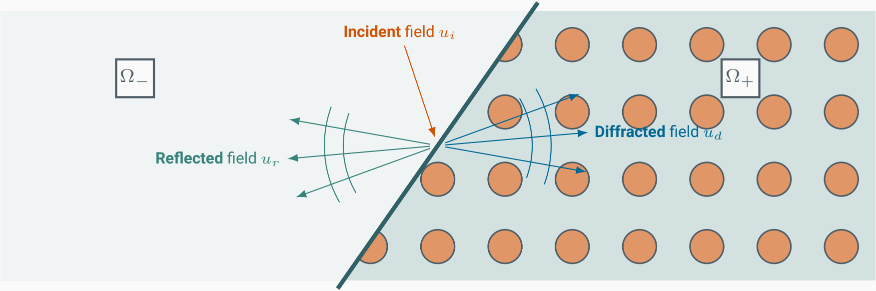
A **first step** is to understand the **half-space** periodic problem.

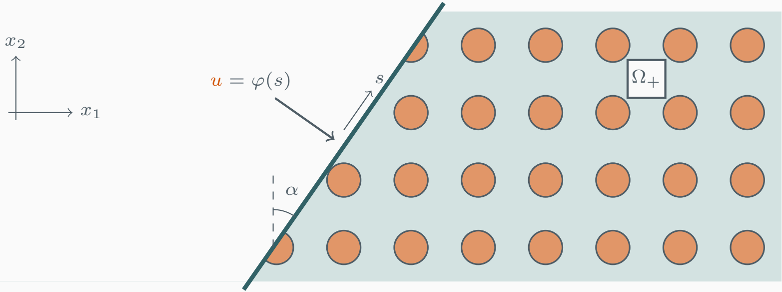


Helmholtz equation with Dirichlet boundary condition

We wish to **compute** $u \in H^1(\Omega_+)$, $\Omega_+ := \{x_1 \tan\alpha - x_2 > 0\}$, the unique solution of

$$\left|\begin{aligned} -\operatorname{div} \mathbf{A}\nabla u - \rho\,\omega^2\, u &= 0, \quad x \in \Omega_+ \\ u &= \varphi, \quad x \in \partial\Omega_+ \end{aligned}\right. \qquad (\mathscr{P})$$

where
- $\mathfrak{Im}\,\omega > 0$
- $\mathbf{A} \in \mathscr{C}^0(\mathbb{R}^2; \mathbb{R}^{2\times 2})$ and $\rho \in \mathscr{C}^0(\mathbb{R}^2)$ are $1-$**periodic**, bounded and coercive
- The Dirichlet data φ belongs to $H^{1/2}(\partial\Omega_+)$.

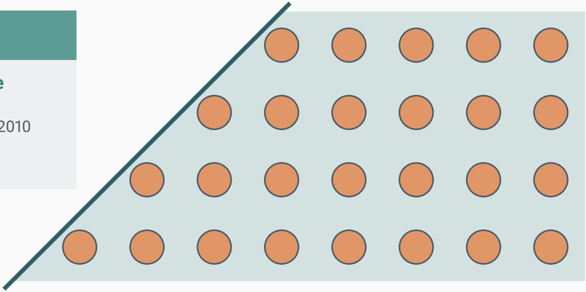


Current methods

When $\tan\alpha$ is **rational**

The medium is **periodic along the interface**
Fliss, *Joly*, 2008; *Fliss*, *Cassan*, *Bernier*, 2010

$\tan\alpha = 1$



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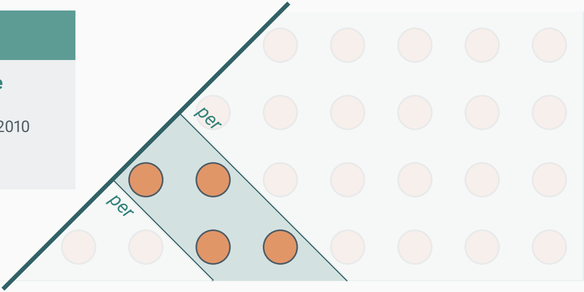

Procedure in the rational case

1. Apply Floquet-Bloch transform in the direction of the boundary
2. Solve a family of waveguide problems parameterized by the Floquet variable

Current methods and limitations

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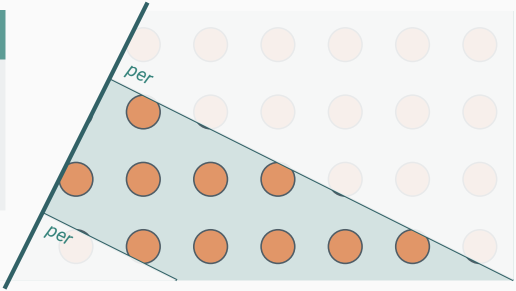
But the domain size **increases** with the **denominator** of $\tan\alpha$.

$$\tan\alpha = 1/2$$



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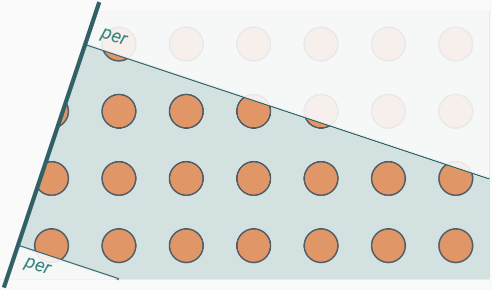

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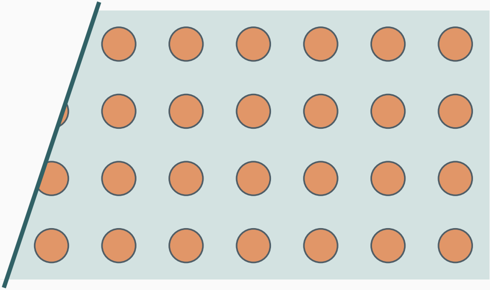
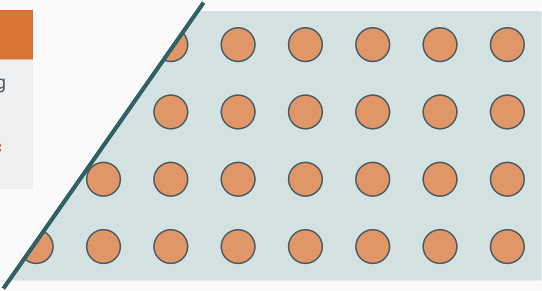


When $\tan \alpha$ is **irrational**

The medium is **no longer** periodic along the interface

But it still has a so-called **quasiperiodic** structure.



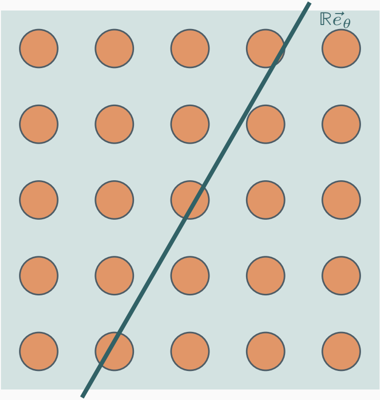
Outline

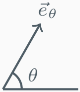
Quasiperiodic functions

Definition – Quasiperiodic functions

A function $\mu_\theta : \mathbb{R} \to \mathbb{C}$ is said to be **quasiperiodic** of order 2 if there exists $\theta \in (0, \pi/2)$ and a continuous function $\mu_p : \mathbb{R}^2 \to \mathbb{C}$, $1-$periodic in each variable, such that

$$\forall\, x \in \mathbb{R}, \quad \mu_\theta(x) = \mu_p(x\, \cos\theta, x\, \sin\theta) = \mu_p(\vec{e}_\theta\, x), \quad \vec{e}_\theta := (\cos\theta, \sin\theta).$$





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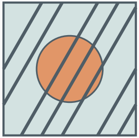

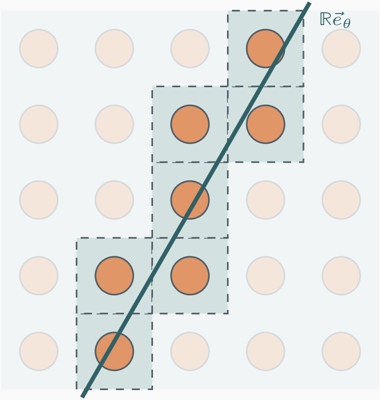

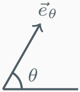

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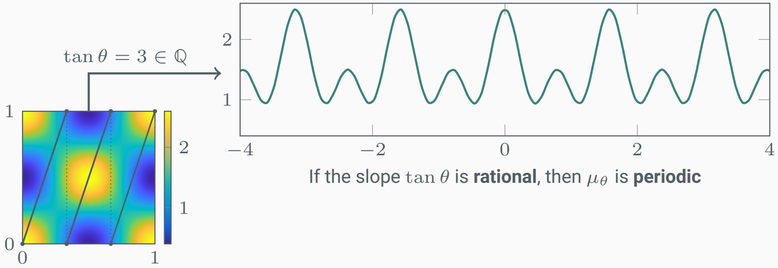

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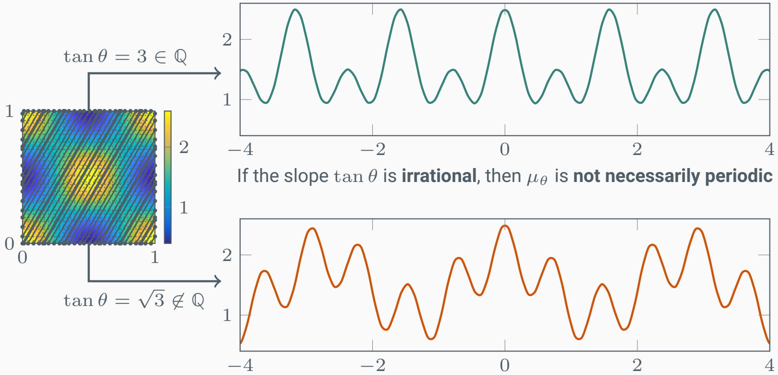
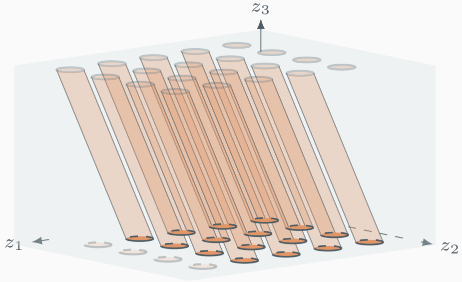


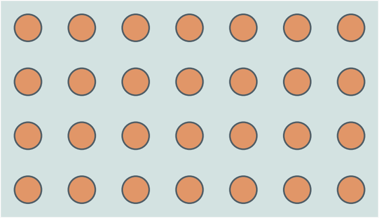
A hidden quasiperiodicity property

Quasiperiodic structure

Define $\forall\ z = (z_1, z_2, z_3) \in \mathbb{R}^3, \quad \mathbf{A}_p(z) = \mathbf{A}(z_1 + z_3, z_2)$ and $\rho_p(z) = \rho(z_1 + z_3, z_2)$

- The tensor $\mathbf{A}_p$ and the coefficient ρ_p are **lifts** of $\mathbf{A}$ and ρ.
- $\mathbf{A}_p$ and ρ_p represent a $1-$**periodic** medium in all directions.





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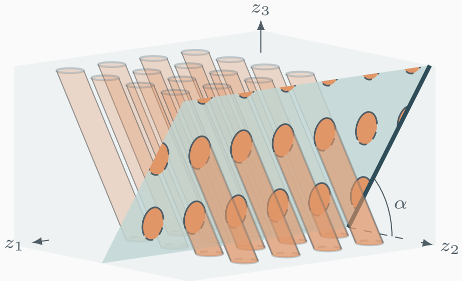
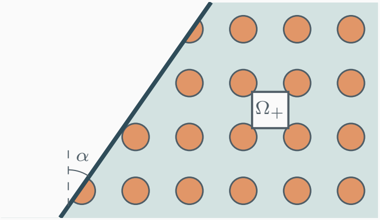
$$\mathbf{C} = \begin{pmatrix} 1 & -\tan\alpha \\ 0 & 1 \\ 0 & \tan\alpha \end{pmatrix}$$





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$$x \in \Omega_+ \iff \mathbf{C}x \in \{(z_1, z_2, z_3) \in \mathbb{R}^3,\ z_1 > 0\}$$
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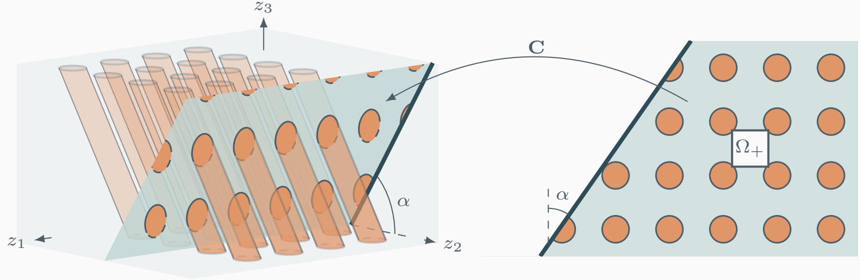

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and

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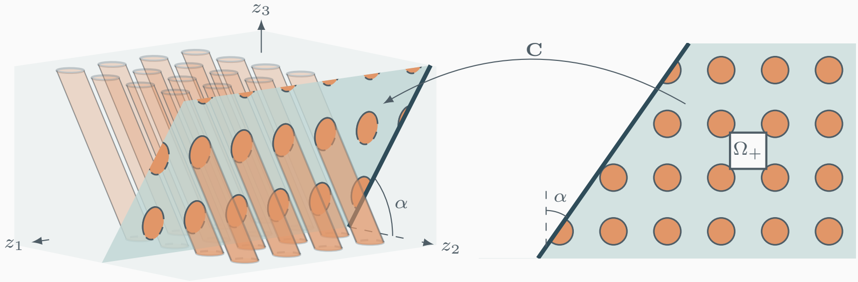

State of the art

Lifting approach

Bouchitté, *Guenneau*, *Zolla*, 2010
Gérard-Varet, *Masmoudi*, 2010
Blanc, *Le Bris*, *Lions*, 2015

Lift the PDE into a non-elliptic PDE with **periodic** coefficients

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State of the art and goal

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Goal of this work

Analysis of wave propagation in quasiperiodicity-induced situations

- 1D Helmholtz equation with quasiperiodic coefficients *A*, *Fliss*, *Joly*, In progress
- Periodic half-space problem

Outline

The lifting approach

Helmholtz equation with Dirichlet boundary condition

We wish to **compute** $u \in H^1(\Omega_+)$, the unique solution of

$$\left| \begin{aligned} -\operatorname{div} \mathbf{A}\nabla u - \rho\,\omega^2\, u = 0, &\quad x \in \Omega_+ \\ u = \varphi, &\quad x \in \partial\Omega_+ \end{aligned} \right. \qquad (\mathscr{P})$$

where $\forall\, x \in \Omega_+, \quad \mathbf{A}(x) = \mathbf{A}_p(\mathbf{C}x)$ and $\rho(x) = \rho_p(\mathbf{C}x)$ with

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and where

$$\mathbf{C} = \begin{pmatrix} 1 & -\tan\alpha \\ 0 & 1 \\ 0 & \tan\alpha. \end{pmatrix} \quad \text{satisfies} \quad x \in \Omega_+ \iff \mathbf{C}x \in \mathbb{R}^3_+ := \{z_1 > 0\}$$

The core idea

Seek u as the trace of a function $U : \mathbb{R}^3_+ \to \mathbb{C}$ along the half-plane $\{\mathbf{C}x,\ x \in \Omega_+\}$, that is,

$$\boxed{u(x) = U(\mathbf{C}\,x).}$$

Lifting onto a periodic half-space problem

Using the ansatz $u(x) = U(\mathbf{C}\, x)$ and the rule $\nabla u(x) = \left[\, {}^t\mathbf{C} \boldsymbol{\nabla} U \right](\mathbf{C}\, x)$, it is natural to introduce

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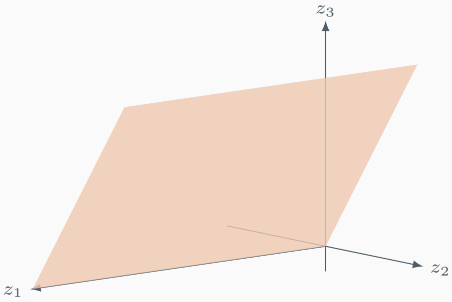

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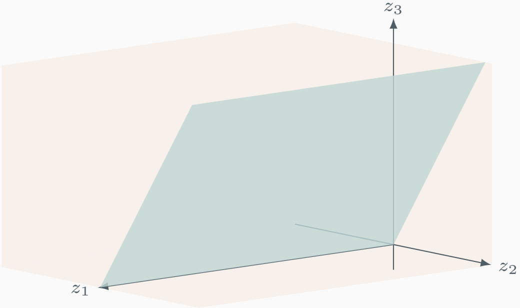

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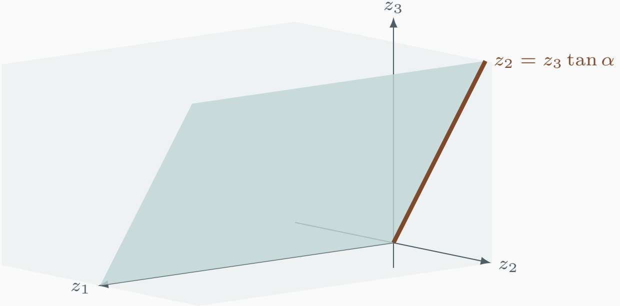

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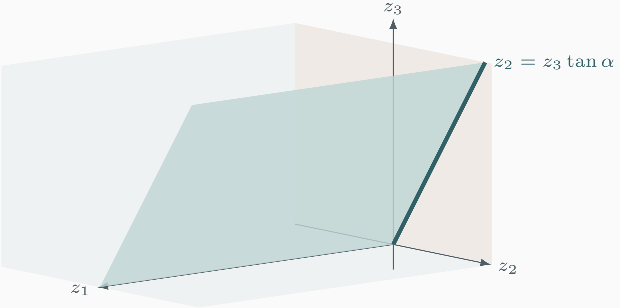

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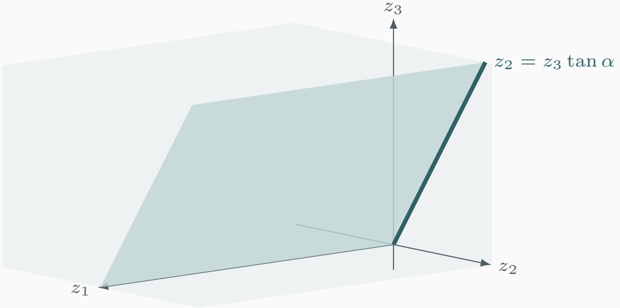

Lifting onto a periodic half-strip problem

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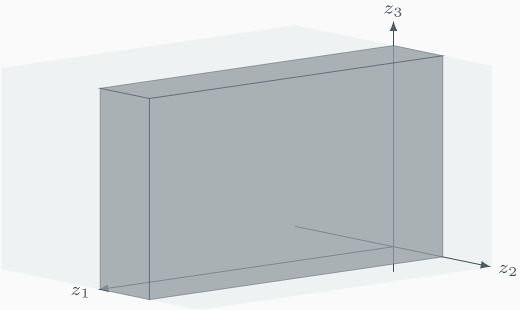
$$\boxed{\varphi_p(z_2 + 1, z_3) = \varphi_p(z_2, z_3) \implies U(\cdot + \vec{e}_2) = U(\cdot).}$$

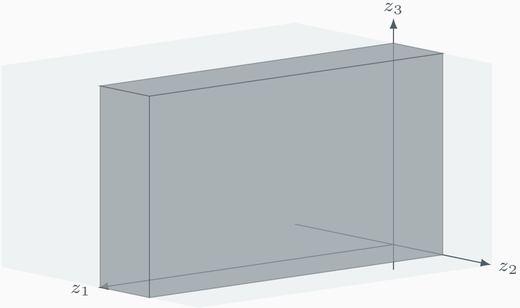


Lifting onto a periodic half-strip problem

Properties of the 3D half-strip problem

- **Periodic** coefficients
- **Nonelliptic** principal part

$$
\begin{aligned}
-\,\mathbf{div}\big(\mathbf{C}\mathbf{A}_p\,{}^{t}\mathbf{C}\nabla U\big) - \rho_p\,\omega^2\,U = 0, &\qquad z_1 > 0,\\
U = \varphi_p, &\qquad z_1 = 0\\
U(\cdot + \vec{e}_2) = U(\cdot) &
\end{aligned}
$$



Resolution of the half-strip problem

How to solve the 3D half-strip problem

Fliss, *Cassan*, *Bernier*, 2010

1. Apply Floquet-Bloch transform along the z_3−axis

$$
\boxed{
\begin{aligned}
-\,\mathbf{div}\left(\mathbf{C}\mathbf{A}_p\,{}^t\mathbf{C}\nabla U\right) - \rho_p\,\omega^2\,U &= 0, \qquad z_1 > 0,\\
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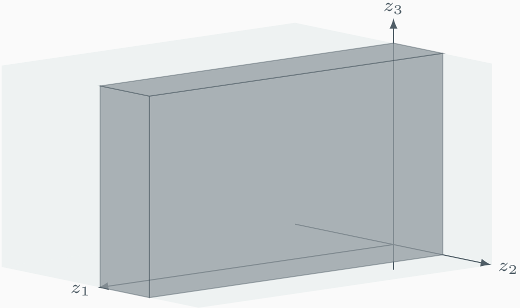
}
$$



Resolution of the half-strip problem

How to solve the 3D half-strip problem

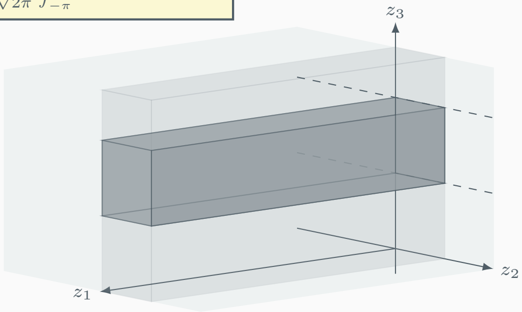
Fliss, *Cassan*, *Bernier*, 2010

1. Apply Floquet-Bloch transform along the z_3−axis
2. Solve a family of waveguide problems parameterized by the Floquet variable k

$$\begin{aligned} -\,\mathbf{div}\big(\mathbf{C}\mathbf{A}_p\,{}^{t}\mathbf{C}\nabla U\big) - \rho_p\,\omega^2\,U &= 0, \qquad z_1 > 0,\\ U &= \varphi_p, \qquad z_1 = 0\\ U(\cdot + \vec{e}_2) &= U(\cdot) \end{aligned}$$

$$\boxed{U(\cdot, z_3) = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \hat{U}_k(\cdot, z_3)\, e^{ikz_3}\, dk}$$

$$\begin{aligned} (\mathbf{div} + ik\,{}^{t}\vec{e}_3)\big(\mathbf{C}\mathbf{A}_p\,{}^{t}\mathbf{C}(\nabla + ik\vec{e}_3)\hat{U}_k\big) - \rho_p\,\omega^2\,\hat{U}_k &= 0\\ \hat{U}_k(\cdot + \vec{e}_3) &= \hat{U}_k(\cdot) \end{aligned}$$



Resolution of the half-guide problem

How to solve the 3D half-strip problem

Fliss, *Cassan*, *Bernier*, 2010

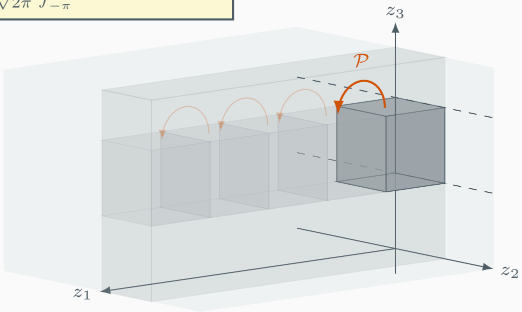
1. Apply Floquet-Bloch transform along the z_3−axis
2. Solve a family of waveguide problems parameterized by the Floquet variable k

$$\left|\begin{aligned} -\,\mathbf{div}\big(\mathbf{C}\mathbf{A}_p\,{}^{\mathrm{t}}\mathbf{C}\nabla U\big) - \rho_p\,\omega^2\,U &= 0, \qquad z_1 > 0,\\ U &= \varphi_p, \qquad z_1 = 0\\ U(\cdot + \vec{e}_2) &= U(\cdot) \end{aligned}\right.$$

$$\boxed{U(\cdot, z_3) = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \hat{U}_k(\cdot, z_3)\, e^{\mathrm{i}kz_3}\, dk}$$

$$(\mathbf{div} + \mathrm{i}k\,{}^{\mathrm{t}}\vec{e}_3)\big(\mathbf{C}\mathbf{A}_p\,{}^{\mathrm{t}}\mathbf{C}(\nabla + \mathrm{i}k\vec{e}_3)\hat{U}_k\big) - \rho_p\,\omega^2\,\hat{U}_k = 0$$

$$\hat{U}_k(\cdot + \vec{e}_3) = \hat{U}_k(\cdot)$$



Half-guide

Fliss, *Joly*, 2008

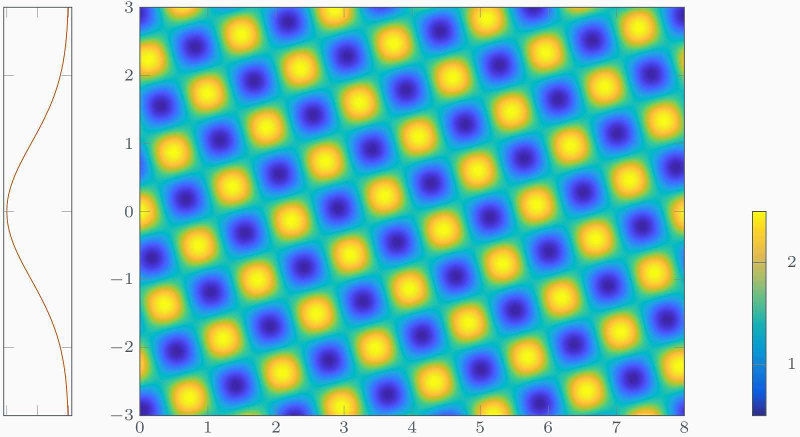
1. Solve local cell problems
2. Compute the propagation operator $\mathcal{P}$

Outline

Numerical results

Test case for the half-space problem

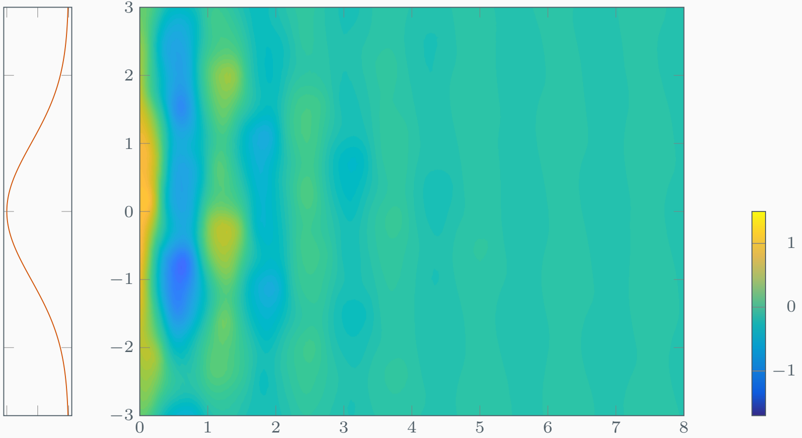
Data φ and function ρ



Numerical results

Test case for the half-space problem

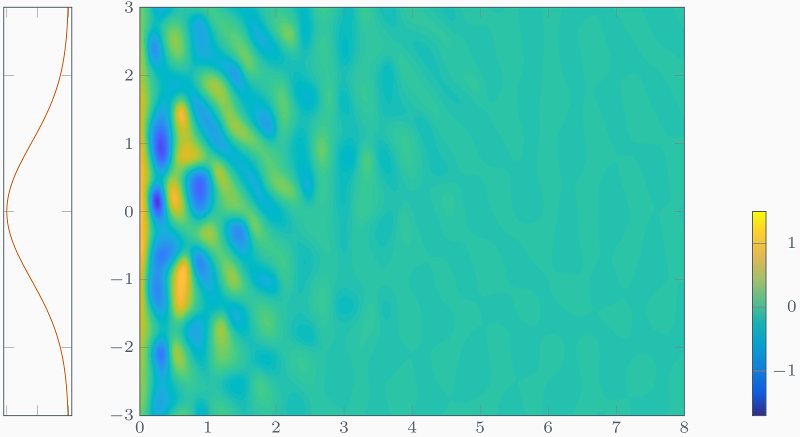
- $\omega = 5 + 0.5\mathrm{i}$
- $\alpha = \pi/3$
- $\varphi(s) = \exp(-s^2/2)$



Numerical results

Test case for the half-space problem

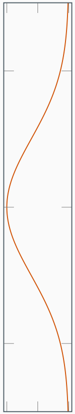
- $\omega = 10 + 0.5\mathrm{i}$ • $\alpha = \pi/3$ • $\varphi(s) = \exp(-s^2/2)$

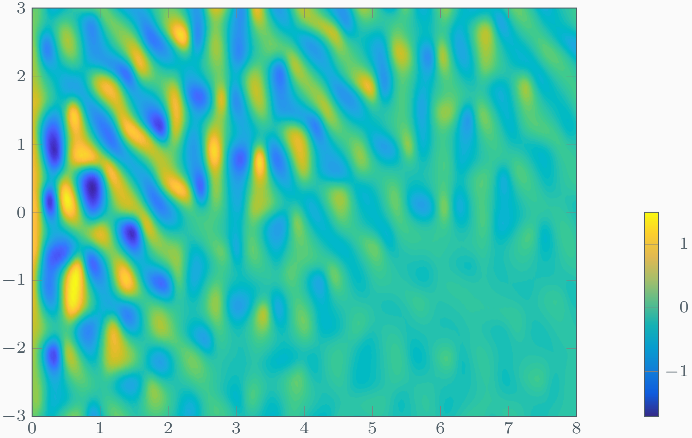


Numerical results

Test case for the half-space problem

- $\omega = 10 + 0.1\mathrm{i}$
- $\alpha = \pi/3$
- $\varphi(s) = \exp(-s^2/2)$





Outline

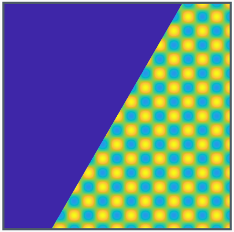
Conclusion

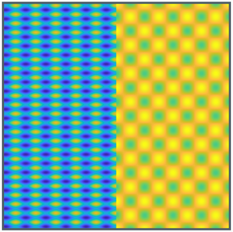
Summary

Resolution of the **Helmholtz equation** in presence of a 2D **periodic halfspace**:
Extend the PDE to a periodic PDE through the **lifting approach**

Perspectives for this work (ongoing)

- Extension to transmission problems





Conclusion

Summary

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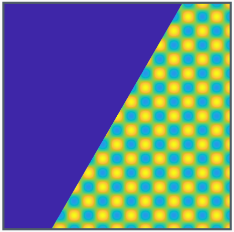

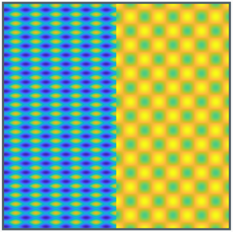

Thank you for your attention!