A Sturm-Liouville problem arising from the study of finite energy GKP qubits

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We study a recent proposal for the autonomous stabilization of a so-called GKP qubit of finite energy [2, 1, 3]. This system is modeled by the Lindblad type equation on $\mathcal{H} = L^2(\mathbf{R}, \mathbf{C})$

$$\frac{d}{dt}\rho = \sum_{k=0}^{4} D[L_k](\rho) \tag{1}$$

where ρ is the *density operator* describing the system, *i.e.* a hermitian, positive semi-definite operator with unit trace on $\mathcal{H}$, $L_k = e^{ik\frac{\pi}{2}\boldsymbol{a}^\dagger\boldsymbol{a}} \left(e^{i\,2\sqrt{\pi}\boldsymbol{Q}}(\mathrm{I} - \epsilon\boldsymbol{P}) - \mathrm{I}\right) e^{-ik\frac{\pi}{2}\boldsymbol{a}^\dagger\boldsymbol{a}}$, $\boldsymbol{a} = \frac{1}{\sqrt{2}}(x+\partial_x)$ is the *annihilation* operator of the harmonic oscillator, $\boldsymbol{Q} = \frac{\boldsymbol{a}+\boldsymbol{a}^\dagger}{\sqrt{2}}$ and $\boldsymbol{P} = \frac{\boldsymbol{a}-\boldsymbol{a}^\dagger}{\sqrt{2}i}$ are the position and impulsion operators, $\epsilon > 0$ is an experimental parameter and $D[L](\rho) := L\rho L^\dagger - \frac{1}{2}\left(L^\dagger L\rho + \rho L^\dagger L\right)$ for any (unbounded) operator L.

Assessing the suitability of this proposal – e.g. the provided protection of the encoded logical information from external perturbations, the experimental parameter range to achieve this protection, etc. – usually requires to simulate this infinite-dimensional system in practically relevant regimes. However, leveraging the fact that the logical information is encoded in the value of observables that are periodic functions of the position and momentum operators, we show that a specific type of perturbation (called *quadrature noise*) induces logical errors whose rates are linked to the first eigenvalues of the Sturm-Liouville operator $\mathcal{L}_\sigma$ defined on 2π-periodic functions by

$$\mathcal{L}_\sigma f(\theta) = \sin(2\theta)f'(\theta) - \sigma f''(\theta) \tag{2}$$

where $\sigma > 0$ is a parameter depending on ϵ and the strength of the quadrature noise. We provide explicit bounds for the first non-zero eigenvalue of $\mathcal{L}_\sigma$ and numerically estimate the following one ; we find close correspondence with the errors rates estimated from a brute force simulation of Equation (1) under quadrature noise.
Additionally, we find that the computed dependence of these error rates on the parameters of the system is still qualitatively correct under a more realistic noise model where the previous Sturm-Liouville correspondence no longer applies.

This work provides a tool to estimate the experimental resources necessary to implement the proposed GKP qubit stabilization scheme. This is joint work with Pierre Rouchon (Mines ParisTech, Inria) and Claude Le Bris (Ponts ParisTech, Inria).

[1] P. Campagne-Ibarcq, et al. *Protected and controllable GKP qubits in high-impedance Josephson circuits*. Oral communication at the APS March Meeting 2022.

[2] D. Gottesman, A. Kitaev, J. Preskill. *Encoding a qubit in an oscillator*. Physical Review A, **64(1)**, 012310. doi :10.1103/PhysRevA.64.012310.

[3] L.-A. Sellem, et al. *In preparation*.

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