

## Noether's-type theorems on time scales

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Dans cet exposé on va prouver la version du théorème de Noether dans le cas time scale dans le cadre calcul des variations sur time scale shifté.

**Théorème 1.** *Let  $\mathbb{T}$  be a time scale such that  $\sigma$  is  $\Delta$ -differentiable on  $\mathbb{T}^\kappa$ . Let  $G = \{T_\epsilon(t, x) = (\phi_\epsilon(t), \psi_\epsilon(x))\}_{\epsilon \in \mathbb{R}}$  be a  $(\Delta, \mathbb{T})$ -variational symmetry of  $\mathcal{L}_{\Delta, \mathbb{T}}^\sigma$  with the corresponding infinitesimal generator given by*

$$X = \xi(t) \frac{\partial}{\partial t} + \eta(x) \frac{\partial}{\partial x}, \quad \xi(t) = \left. \frac{\partial \phi_\epsilon}{\partial \epsilon} \right|_{\epsilon=0}, \quad \eta(x) = \left. \frac{\partial \psi_\epsilon}{\partial \epsilon} \right|_{\epsilon=0}$$

*Then, the quantity*

$$I(t, x^\sigma, v) = -\mathcal{H}(\star_\sigma)\xi(t) + \partial_v L(\star_\sigma)\eta(x) + \int_a^t \xi^\sigma(t) \left( \Delta[\mathcal{H}(\star_\sigma)] + \partial_t L(\star_\sigma) \right) \Delta t \quad (1)$$

*is a constant of motion over the solution of the time scales Euler-Lagrange equation*

$$\Delta \left[ \frac{\partial L}{\partial v}(t, x^\sigma(t), \Delta x(t)) \right] = \frac{\partial L}{\partial x}(t, x^\sigma(t), \Delta x(t)) \quad (2)$$

*meaning that*

$$\Delta [I(\cdot, x^\sigma(\cdot), \Delta x(\cdot))] (t) = 0,$$

*for all solutions  $x$  of (2) and any  $t \in \mathbb{T}^{\kappa^2}$ , where  $(\star_\sigma) = (t, x^\sigma(t), \Delta x(t))$  and  $\mathcal{H}$  is given by*

$$\mathcal{H}(t, u, v) = -L(t, x, v) + \partial_v L(t, x, v) \cdot v + \partial_t L(t, u, v) \mu(t).$$

Voir la preuve dans [1].

[1] B. Anerot, J. Cresson, K. H. Belgacem, F. Pierret3. Noether's-type theorems on time scales. Journal of Mathematical Physics, **61**(11), 2020.