Influence des caractéristiques sociales dans les modèles épidémiques SIR

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ANR SPACE-Covid

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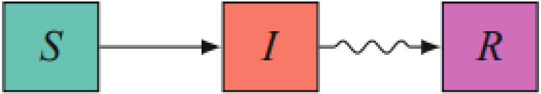
- I. SIR models
- II. Metapopulation models
  - Contact matrix
  - Application to age-structured population
- III. Change of representation
  - Subdivision of population into social classes

The SIR model

Subdivision of population in 3 compartments

- Susceptibles
- Infected
- Recovered



The SIR model

Population of size N constant.

$$\frac{dS(t)}{dt} = -S(t)F(t)$$
$$\frac{dI(t)}{dt} = S(t)F(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

with $S(t) + I(t) + R(t) = N, \forall t > 0$ and with $F(t) = \lambda \frac{I(t)}{N}$ the *infection force*.
In the following, we set $\lambda = \beta k$.

Basic reproduction number R_0

Assuming $S(0) \simeq N$ and $I(0) = 1$

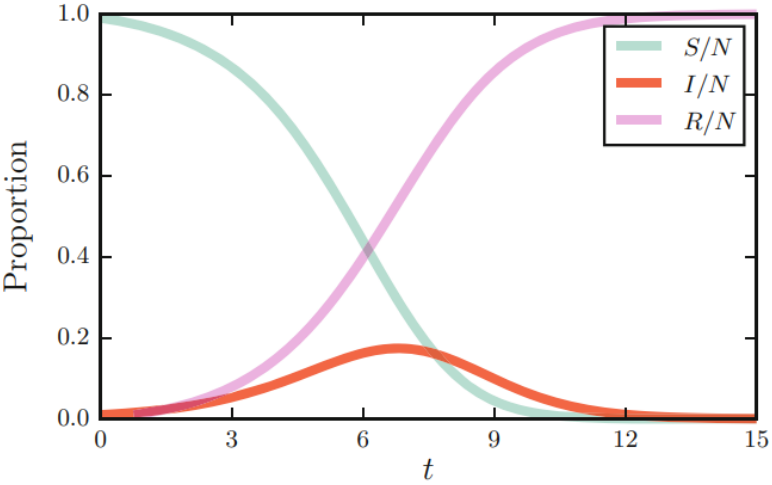
$$\left.\frac{dI(t)}{dt}\right|_{t=0} > 0 \iff \lambda \frac{S(0)I(0)}{N} - \gamma I(0) > 0 \tag{1}$$

$$\iff \frac{\lambda}{\gamma} > 1 \tag{2}$$

Definition

$R_0 = \frac{\lambda}{\gamma}$

$R_0 > 1$



New compartments

$$\frac{dS}{dt} = -SF \tag{3}$$

$$\frac{dE}{dt} = SF - \delta E \tag{4}$$

$$\frac{dI^a}{dt} = (1-p)\delta E - \gamma I^a - \mu I^a \tag{5}$$

$$\frac{dI^s}{dt} = p\delta E - \gamma I^s - \mu I^s \tag{6}$$

$$\frac{dR}{dt} = \gamma(I^a + I^s) \tag{7}$$

$$\frac{dD}{dt} = \mu(I^a + I^s) \tag{8}$$

Infection force: $F = \frac{\beta}{N} k(I^a + fI^s)$

Hypothesis of compartmental models

- well-mixed
- homogenous

II. Metapopulation SEIR model

Incorporing heterogeneity : metapopulation

Subdivision of population of size N into n classes.
In class i, we have

$$\frac{dS_i}{dt} = -S_i F_i \tag{9}$$

$$\frac{dE_i}{dt} = S_i F_i - \delta E_i \tag{10}$$

$$\frac{dI_i^a}{dt} = (1-p)\delta E_i - \gamma I_i^a - \mu I_i^a \tag{11}$$

$$\frac{dI_i^s}{dt} = p\delta E_i - \gamma I_i^s - \mu I_i^s \tag{12}$$

$$\frac{dR_i}{dt} = \gamma (I_i^a + I_i^s) \tag{13}$$

$$\frac{dD_i}{dt} = \mu_i (I_i^a + I_i^s) \tag{14}$$

Infection force: $F_i = \frac{\beta}{N} \sum_{j=1}^{n} \kappa_{ij} (I_j^a + f I_j^s)$

Contact matrix κ

Data available from surveys of sizes N_{samp}

- B sampled *raw* matrix of contacts of size $n \times n$. The coefficient b_{ij} is the total number of contacts between participants of the survey of class i and one of their contacts belonging to class j.
- $\pi = (\frac{N_i}{N})_{1 \leq i \leq n}$ relative proportions of the classes in the population.

Let k the average number of contacts

$$k_i = \frac{\sum\limits_{j=1}^{n} b_{ij}}{N_{samp,i}}. \tag{15}$$

Let C the stochastic matrix

$$c_{ij} = \frac{b_{ij}}{\sum\limits_{j=1}^{n} b_{ij}}. \tag{16}$$

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Definition

From C, k and π, we build κ s.t. $\kappa_{ij} = \frac{c_{ji} k_j}{\pi_i}$.

Example

Subdivision into young (Y) and old (O).

- $\pi = (0.2, 0.8)$
- $N_{samp} = 1200,\ N_{samp,Y} = 1000,\ N_{samp,O} = 200$
- $B = \begin{pmatrix} 6000 & 4000 \\ 100 & 900 \end{pmatrix}.$

From B, $N_{samp,Y}$ and $N_{samp,O}$ we derive

$$k = \left(\frac{10000}{N_{samp,Y}}, \frac{1000}{N_{samp,O}}\right) = (10, 5). \tag{18}$$

and

$$C = \begin{pmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{pmatrix}. \tag{19}$$

We then construct κ using C, k and π. We have

$$\kappa = \begin{pmatrix} 0.6 \times 10/0.2 & 0.1 \times 5/0.2 \\ 0.4 \times 10/0.8 & 0.9 \times 5/0.8 \end{pmatrix} = \begin{pmatrix} 30 & 2.5 \\ 5 & 5.625 \end{pmatrix} \tag{20}$$

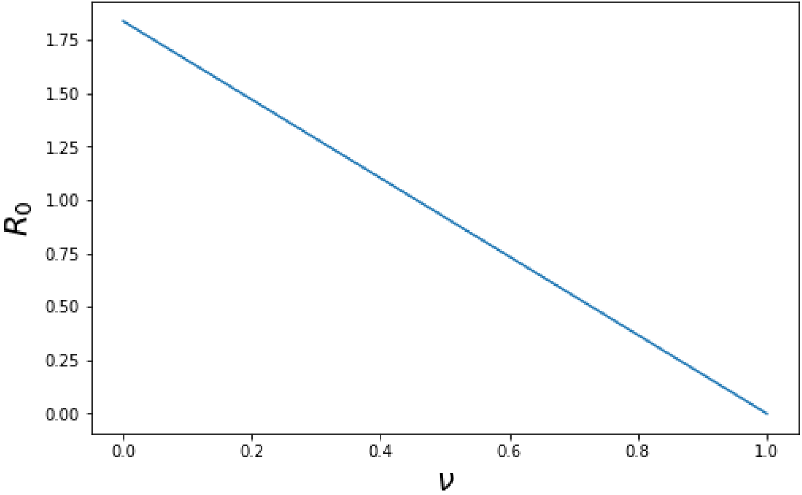
Age-structured SIR - Numerical results

- French contact matrix from [1]
- $N = 65000000$
- π from National Institute of Statistics and Economic Studies (https://statistiques-locales.insee.fr/)
- Infection Fatality Ratio μ from [2]

Proportion of initial infected $P = 0.001$ evenly distributed among the different classes.
Population is subdivided into three age groups: 0 to 17, 18 to 64 and 65 or older.

Impact of containment on R_0

$k_\nu = (1 - \nu)k$



Limitation of contacts within a class

We remove a fraction ν of the contacts made by a class i_0. To do this, we construct a *raw* contact matrix B_ν such that

$$b_{\nu,ij} = (1-\nu)b_{ij},\ \forall j, i = i_0 \tag{21}$$

$$b_{\nu,ij} = b_{ij},\ \forall i, j \neq i_0. \tag{22}$$

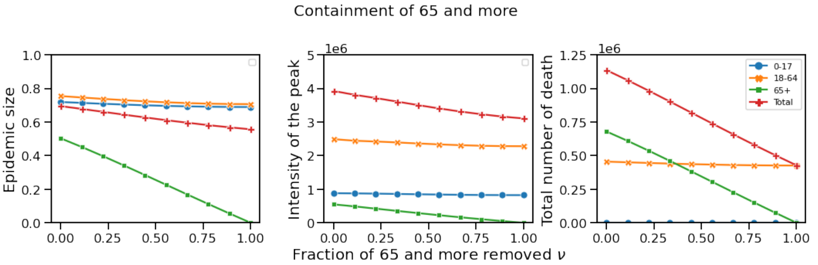
Moreover, we pass from the vector of average contacts k to a vector k_ν such that

$$k_{\nu,i} = k_i - \nu k_i c_{i,i_0},\ \forall i \neq i_0, \tag{23}$$

$$k_{\nu,i_0} = k_{i_0}(1-\nu). \tag{24}$$

We then obtain from B_ν, k_ν et π a contact matrix κ_ν.

Containment of older



III. Change of representation : from age classes to social classes

Social contact matrix. From σ to τ

We have B_σ of size $n \times n$, created from classes $(\sigma_i)_{1 \le i \le n}$. We want B_τ describing the contacts between classes $(\tau_i)_{1 \le i \le m}$.

Definition

Let be subdivisions $\sigma = (\sigma_i)_{1 \le i \le n}$ and $\tau = (\tau_i)_{1 \le i \le m}$ of the population.
We call *change of representation matrix* the matrix R of size $n \times m$ where r_{ij} is the probability that an individual of σ_i belongs to τ_j.
We construct a *raw* contact matrix B_τ for the classes $(\tau_i)_{1 \le i \le m}$ in the following manner

$$B_\tau = R^T B_\sigma R. \tag{25}$$

Social contact matrix. From τ to σ

Remark

Let R a *change of representation matrix* between classes σ and τ and π relative size of classes σ.
An application of Bayes formula gives $\tilde{R}$, the *change of representation matrix* from class τ to σ.

$$\tilde{r}_{ij} = \frac{r_{ji}\pi_j}{\sum_{1 \le k \le n} r_{ki}\pi_k} \tag{26}$$

Proposition

Let R and $\tilde{R}$ matrices of changes in representations between classes σ and τ. Let B a *raw* contact matrix. If the product $R\tilde{R}$ is irreducible and aperiodical, then the sequence $((R\tilde{R})^T B(R\tilde{R}))_n$ has a limit.

Information loss

We quantify information loss with

$$\|Id - (R\tilde{R})\|_2 \tag{27}$$

Example - no information loss

We want to pass from (σ_1, σ_2) to (τ_1, τ_2, τ_3). Let π the relative proportions of σ in the population.
Suppose two different age classes cannot be in the same τ_i class.
For example

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix}. \tag{28}$$

In this case, the matrix $\tilde{R}$ allowing to pass from τ to σ is

$$\tilde{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}. \tag{29}$$

So $R\tilde{R} = I_2$ and $\|I_2 - R\tilde{R}\| = 0$.

Example - loss of information

Let us suppose now that for all i an individual of the class σ_i has as much chance to belong to each class $\tau_j, 1 \leq j \leq m$. We then have $r_{ij} = \frac{1}{m}$ and

$$R\tilde{R} = \begin{pmatrix} \pi_1 & \pi_2 \\ \pi_1 & \pi_2 \end{pmatrix}. \tag{30}$$

Hence $\|I_2 - R\tilde{R}\| = \left((1-\pi_1)^2 + (1-\pi_2)^2 + \pi_1^2 + \pi_2^2\right)^{\frac{1}{2}}$.
Starting with *raw* contact matrix B_σ and applying successively the transformation to τ and then the transformation to σ, we obtain

$$(R\tilde{R})^T B_\sigma (R\tilde{R}) = \sum_{i,j} b_{ij} \begin{pmatrix} \pi_1^2 & \pi_1\pi_2 \\ \pi_1\pi_2 & \pi_2^2 \end{pmatrix}. \tag{31}$$

Numerical results - containment of white collars

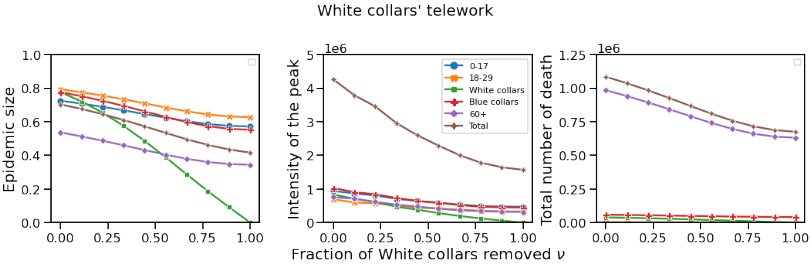
Based on 4 age classes, the population is subdivided into 5 social classes.

	0-17	18-29	White collar	Blue collar	60+
0-17	1	0	0	0	0
18-29	0	1	0	0	0
30-60	0	0	0.45	0.55	0
60+	0	0	0	0	1

Table: Representation change matrix used for white collar telework

$$\|Id - R\tilde{R}\|_2 = 0$$

Numerical results - containment of white collars



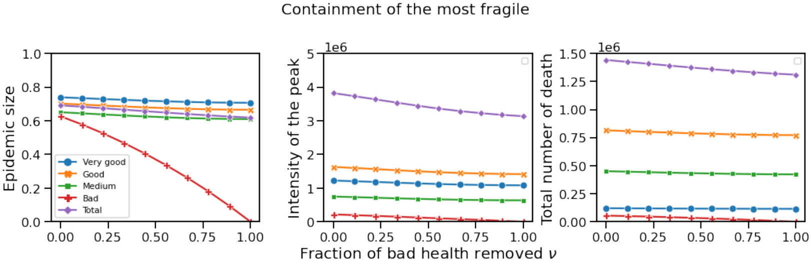
Numerical results - Containment of the most fragile

	Very good	Good	Medium	Bad
0-15	0.50	0.40	0.08	0.02
16-29	0.50	0.40	0.08	0.02
30-49	0.30	0.47	0.18	0.05
50-74	0.13	0.47	0.32	0.08
75+	0.04	0.28	0.46	0.22

Table: Representation change matrix used for containment of the most fragile

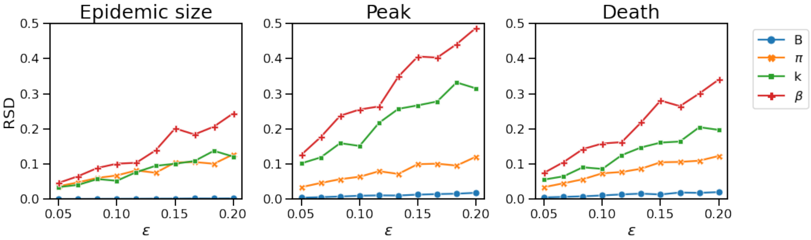
$$\|Id - R\tilde{R}\|_2 = 1.93$$

Numerical results - Containment of the most fragile



Parameters sensitivity

For each input x tested, we run 100 simulations. At each simulation the parameter follows a normal distribution $\mathcal{N}(x, \varepsilon)$.



Conclusion and perspectives

+ metapopulation : incorporing heterogeneity without intractabilty
+ exploring new subdivisions of population
- depending on the notion of contact

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+ metapopulation : incorporing heterogeneity without intractabilty
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- reinfection
- mutation
- time dependance of κ
- sanitary measures

Merci de votre attention.

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Closure of schools

Based on 7 age classes, the population is subdivided into 6 social classes.

Scenario	Relative size	Number of contacts	Epidemic size	Peak	Number of Death
Kindergarten	0.03	10.1			
Elementary	0.06	10.8			
Middle school	0.06	12.8			
High school	0.03	14.3			
University	0.08	10			

Table: Decrease (within the total population) of the various indicators between a simulation without contact limitation and a maximum limitation according to the chosen scenario

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Scenario	Relative size	Number of contacts	Epidemic size	Peak	Number of Death
Kindergarten	0.03	10.1	0.06	0.08	0.03
Elementary	0.06	10.8	0.11	0.16	0.07
College	0.06	12.8	0.12	0.24	0.07
High school	0.03	14.3	0.08	0.16	0.05
University	0.08	10	0.16	0.27	0.12

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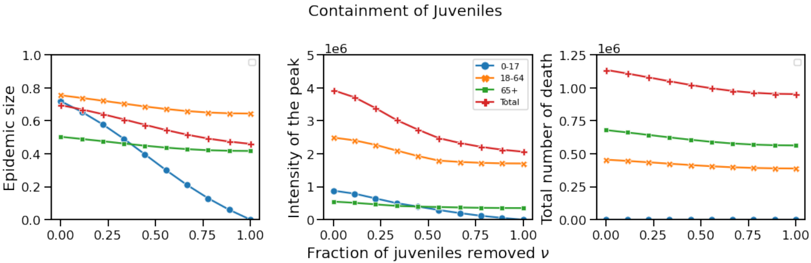
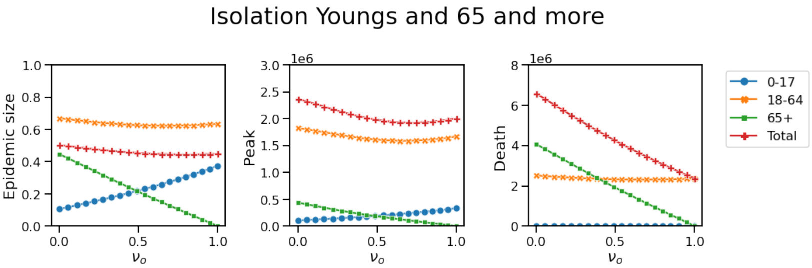
Containment of juvenils



Limitation of youngs and olders

$\nu = 0.25$ of removed contacts within total population We remove in Youngs and Old



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