Variational discrete approximation of the Griffith functional by adaptative finite elements

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This work is devoted to show a discrete adaptative finite element approximation result for the isotropic two-dimensional Griffith energy arising in fracture mechanics. The problem is addressed in the geometric measure theoretic framework of generalized special functions of bounded deformation which corresponds to the natural energy space for this functional. It is proved to be approximated in the sense of Γ-convergence by a sequence of integral functionals defined on continuous piecewise affine functions. The main feature of this result is that the mesh is part of the unknown of the problem, and it gives enough flexibility to recover isotropic surface energies.

More precisely, let us consider a linearly elastic material, whose reference configuration is Ω, a bounded open set of $\mathbb{R}^2$ with Lipschitz boundary. We consider the finite element space $V_\varepsilon(\Omega)$ of all continuous displacements $u : \Omega \to \mathbb{R}^2$ for which there exists an admissible trianguation $\mathbf{T}$ of Ω such that u is affine on each triangle $T \in \mathbf{T}$.

Our main result is the following Γ-convergence approximation of the Griffith functional :

Théorème 1. *The brittle damage energy defined for displacements $u \in V_\varepsilon(\Omega)$ by*

$$\int_\Omega \frac{\kappa}{\varepsilon} \wedge |e(u)|^2 \, dx$$

Γ-converges, with respect to the $L^0(\Omega;\mathbb{R}^2)$-topology of convergence in measure, to the Griffith functional

$$\mathcal{F}(u) = \int_\Omega |e(u)|^2 \, dx + \kappa \sin\theta_0 \mathcal{H}^1(J_u),$$

where the toughness of the limit material, $\kappa \sin\theta_0$, is explicit and only depends on κ and the geometry of our admissible triangulations, as θ_0 stands for the minimal angle of the admissible triangles.

This is joint work with Jean-François Babadjian.

(Article submitted)

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