A well-balanced entropy scheme for a shallow water type system describing two-phase debris flows

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In the context of modeling two-phase debris flows involving grains and fluid such as shown on 1 [1], some shallow water systems arise with internal variables. Our work focus on such a shallow water system (1), (2) with two internal variables (3), (4) and a topography b which adds a nonconservative term.

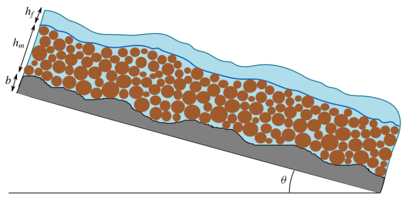

FIGURE 1 – Two-phase two-layers grain and fluid flow

For numerical purposes, it is desirable to deal with a system where the mathematical entropy (the physical energy of the system (5)) is convex with respect to the chosen conservative variables. The computation also imposes conditions on the internal variable ρ. Then at the numerical level, we can look for a scheme satisfying a semi-discrete entropy inequality.

Moreover a crucial point in modeling debris flows is to well describe the stopping of the flow. In particular, the flow should stop when it is a steady state at rest, which is called well-balanced property.

Our system is written as

$$\partial_t h + \nabla_{\mathbf{x}} \cdot (hv) = 0, \tag{1}$$

$$\partial_t(hv) + \nabla_{\mathbf{x}} \cdot \Big(hv \otimes v\Big) + g_c \nabla_{\mathbf{x}}\Big(r\frac{h^2}{2}\Big) + g_c h \nabla_{\mathbf{x}}(b + \tilde{b}) = T, \tag{2}$$

$$\partial_t \rho + v \cdot \nabla_{\mathbf{x}} \rho = \Phi_1, \tag{3}$$

$$\partial_t r + v \cdot \nabla_{\mathbf{x}} r = \Phi_2, \tag{4}$$

with the energy

$$E = h\frac{|v|^2}{2} + g_c h(b + \tilde{b}) + g_c r \frac{h^2}{2}. \tag{5}$$

The physical unknowns of the system are the total mass h, the velocity v, the density of the mixture layer ρ and a variable r depending on the proportion of fluid between the layers. Sources terms Φ_1, Φ_2 and T contains multivalued friction and dilatancy effects.

Writing the system with conservative variables for which the energy is convex, we derive a well-balanced scheme satisfying a semi-discrete entropy inequality.

[1] F. Bouchut, E. D. Fernández-Nieto, A. Mangeney, G. Narbona-Reina. *A two-phase two-layer model for fluidized granular flows with dilatancy effects.* Journal of Fluid Mechanics, **801**, 166–221, 2016.

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