Topology optimization of structures and additive manufacturing: the design of supports

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Outline

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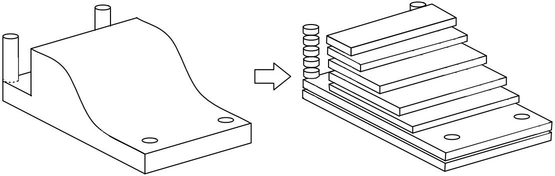
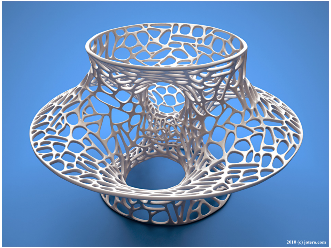
SOFIA: SOlutions pour la Fabrication Industrielle Additive métallique (Add-Up, Michelin, Safran, ESI, etc.).

I - Additive Manufacturing (a.k.a. 3-d printing)

- Structures built layer by layer

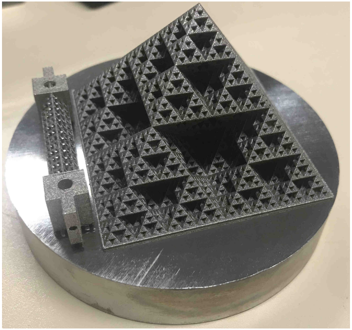


- No topological constraints on the built structures

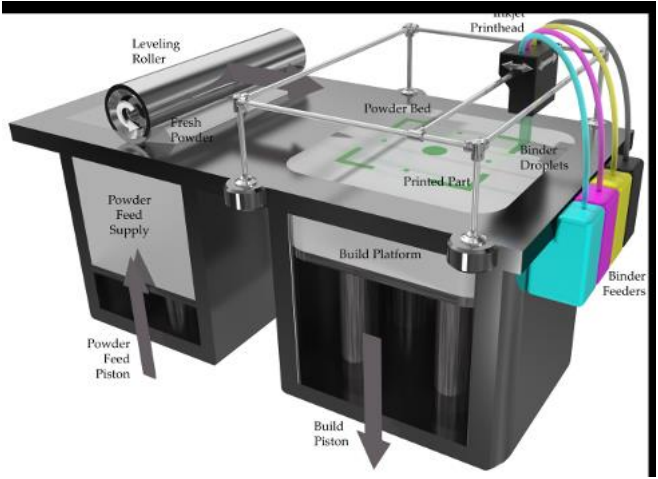


Additive manufacturing

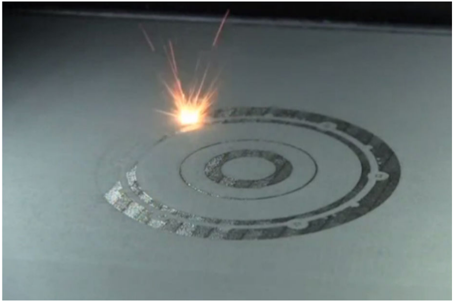
- Various materials: plastic, polymer, metal, ceramic...
- We focus on metallic additive manufacturing
- Various processes: wire, direct energy deposition (DED), layer by layer...
- We focus on powder bed techniques

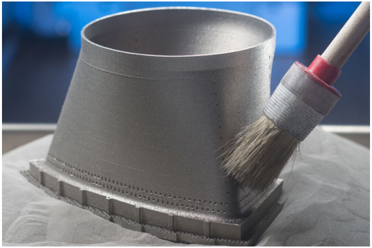


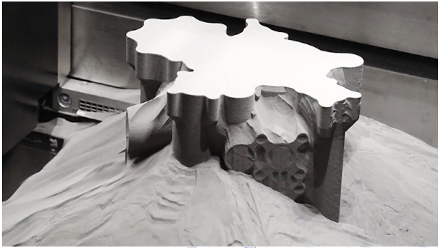
Metallic powder melted by a laser or an electron beam.



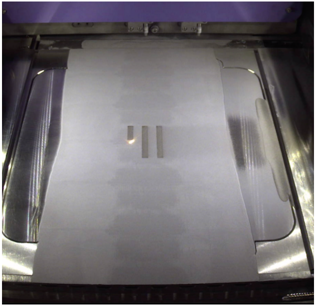
Metallic additive manufacturing



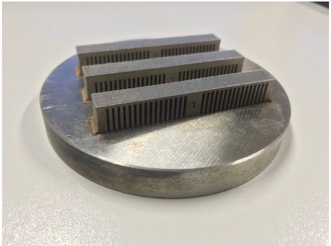


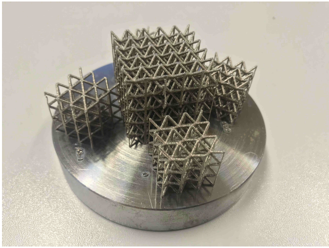


FormUp
AddUp
AddUp

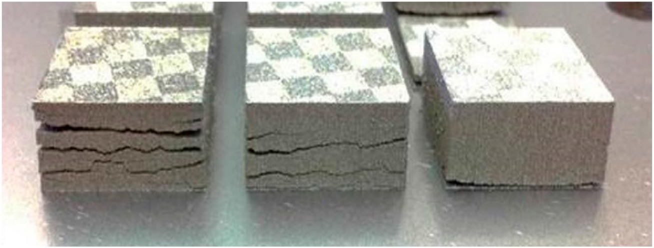


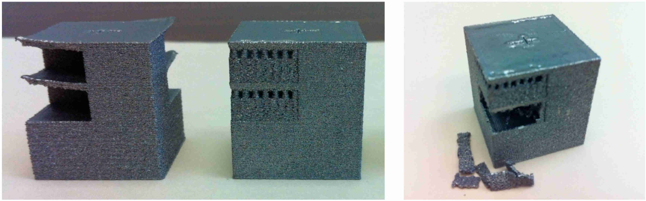
- Very different from classical techniques (molding, milling)
- No topological constraints on the built structures
- Very complicated structures: new applications, new designs
- Possible failures: new constraints !



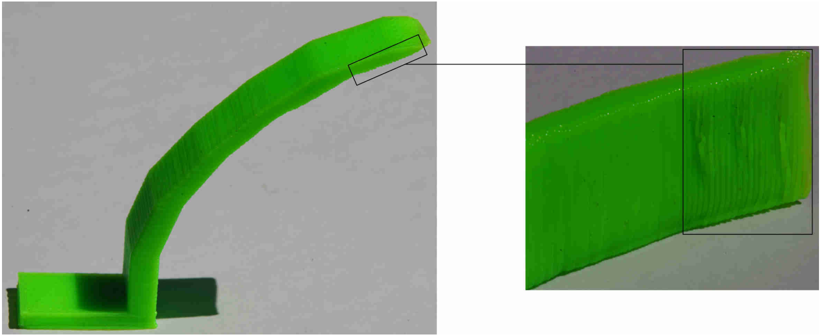


Thermal stresses and deformations:





Other failure: overhang limitation



The angle between the structural boundary and the build direction has an impact on the quality of the processed shape.

Constraints in Additive Manufacturing

Constraints are required to avoid failures in the fabrication process.

Two typical constraints:

- avoid **overhangs** or almost horizontal surfaces (which cannot be built),
- avoid **thermal deformations** or thermal residual stresses which are caused by the high temperatures due to metal melting.

There are other constraints, not discussed here.

Two constraints related to the fabrication process:

- almost horizontal overhang surfaces cannot be realized directly
- thermal residual stresses and thermal deformations

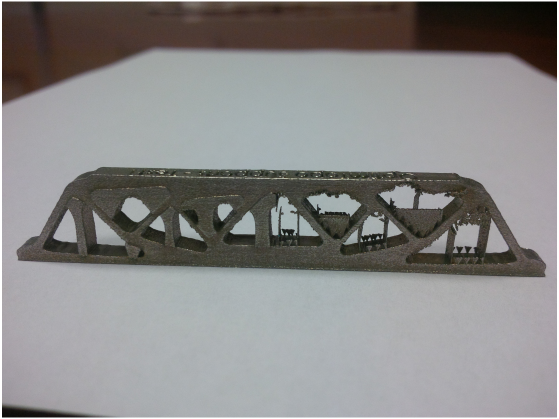
Supports can help:

- they support inclined surfaces
- they fix the shape to the baseplate

Supports can be full material or a lattice (perforated) material.

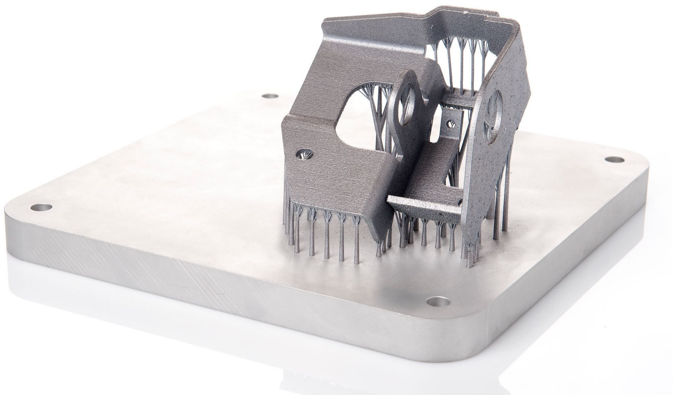
Example 2 of supports

The interface between supports and the part to built can be pre-cut (courtesy of M. Bihr, Safran).



Example 3 of supports

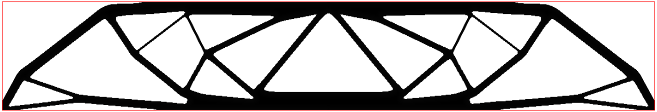
The supports can have a tree structure (Magics ®).



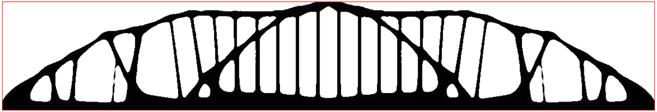
Supports have an additional cost and are not easy to remove.

Why not avoiding supports by optimizing the structure to be self-supported ?

Compliance-optimal MBB beam



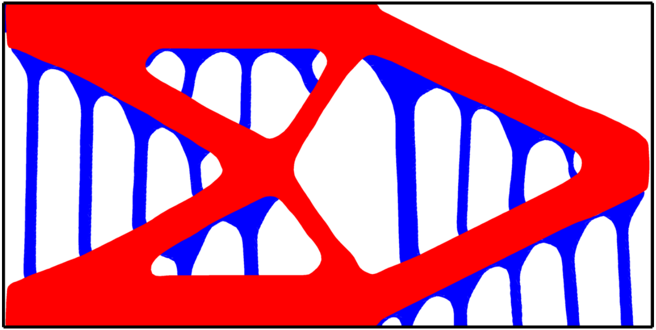
Self-supported-optimal MBB beam



Allaire et al., JCP 351, 295-328 (2017).

Unfortunately, sometimes the design is not allowed to be changed to make it self-supported.

- design domain D (here, a rectangle)
- given structure $\omega \subset D$ (in red) to be printed and not optimizable
- supports $S \subset D$ (in blue) to be optimized
- mechanical model in $\Omega = \omega \cup S$
- objective function to mitigate overhangs



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Many works !

Allaire et al., C. R. Math. Acad. Sci. Paris (2017), Cacace et al., Appl. Math. Model. (2017), Calignano, Materials & Design (2014), Dumas et al., ACM Trans. Graph. (2014), Gaynor and Guest, SMO (2016), Hu et al., Computer-Aided Design (2015), Kuo et al., SMO (2018), Langelaar, Additive Manufacturing (2016), Leary et al., Materials & Design (2014), Mirzendehdel and Suresh, Computer-Aided Design (2016), Qian, J. Num. Meth. Eng. (2017), Strano et al., Int. J. Adv. Manufact. Techn. (2013), Vanek et al., Computer Graphics Forum (2014), etc.

Allaire, Bogosel, Optimizing supports for additive manufacturing, SMO (2018).

Typical formulation

$$\text{Minimize } J(S),$$

where $J(S)$ is related to the rigidity of the total shape $\Omega = S \cup \omega$.

- the structure ω is fixed and only the support S is optimizable
- the state equation is posed in the union $S \cup \omega$
- the material parameters may be different in ω and S
- Forces: model the "instability" of inclined regions
- Volume constraint for the support $\text{Vol}(S)$

State equation: linearized elasticity

Pseudo-gravity loads, parallel to the build direction, in ω and S:

$$\begin{cases} -\operatorname{div}\sigma &= g(\rho_\omega\chi_\omega + \rho_S\chi_S) & \Omega = \omega\cup S \\ \sigma &= 2\mu e(u) + \lambda\operatorname{div} u\,\mathrm{Id} & \Omega \\ e(u) &= \frac{1}{2}(\nabla u + \nabla^t u) & \Omega \\ u &= 0 & \Gamma_D \\ \sigma.n &= 0 & \Gamma_N \end{cases}$$

Compliance minimization:

$$J(S) = \int_{\omega\cup S} g(\rho_\omega\chi_\omega + \rho_S\chi_S)\cdot u$$

where χ_ω and χ_S are the characteristic functions of ω and S.
Typically $\rho_S = 0$.

Hadamard setting: the support S is perturbed by a vector field $\theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$

$$\theta \mapsto S_\theta := (Id + \theta)(S) = \{x + \theta(x) : \ x \in S\}$$

Because S is restricted to belong to $D \setminus \omega$, the set of admissible deformations is defined as

$$\Theta_{ad} = \left\{\theta \in W^{1,\infty}(D, \mathbb{R}^d) : \ \|\theta\|_{W^{1,\infty}} < 1, \ \theta \cdot n = 0 \text{ on } \partial D \cup \partial\omega\right\}.$$

Definition. A function $J(S)$ is shape differentiable if the map $\theta \in \Theta_{ad} \mapsto J(S_\theta)$ is Fréchet-differentiable at 0

$$J(S_\theta) = J(S) + J'(S)(\theta) + o(\theta), \text{ with } \lim_{\theta \to 0} \frac{|o(\theta)|}{\|\theta\|_{W^{1,\infty}}} = 0.$$

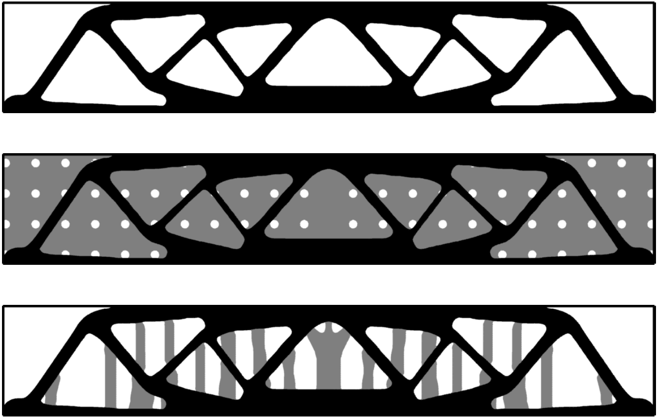
Theorem. The compliance $J(S)$ is shape differentiable and its derivative is

$$J'(S)(\theta) = \int_{\partial S \setminus \partial\omega} \left(-Ae(u)\cdot e(u) + 2\rho g \cdot u\right) \theta \cdot n \, ds$$

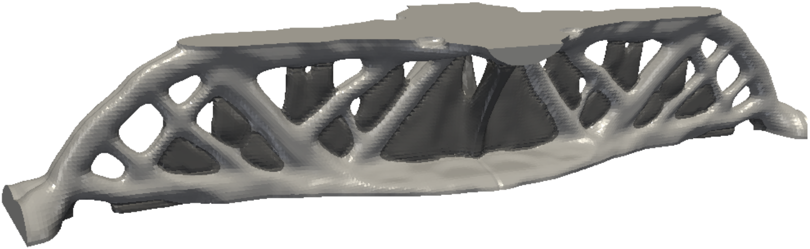
Numerical method (very classical by now):

- the support S is represented by a level set function
- the shape derivative is used for advecting the level set
- an augmented Lagrangian algorithm allows to take into account constraints

MBB beam in 2D (supports in grey)

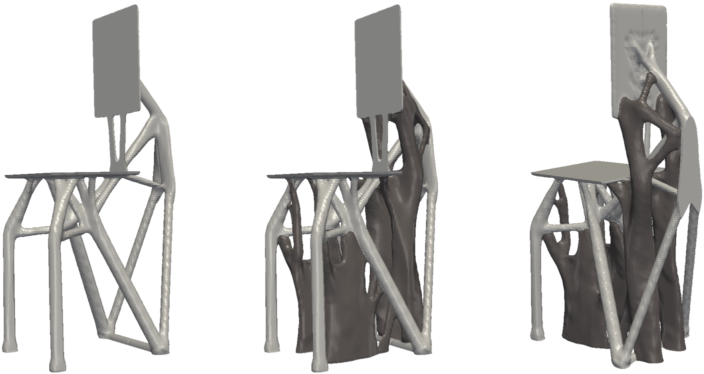


MBB beam in 3D



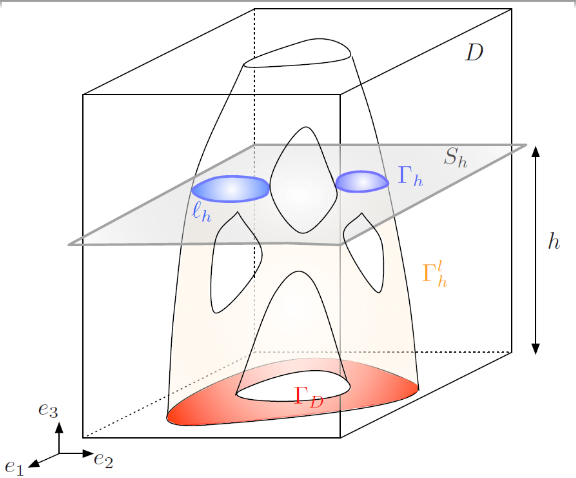
Rotation

Chair in 3D



Rotation   Optimization

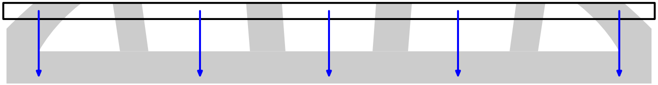
Layer by layer modeling



For a final shape $\Omega = \omega \cup S$, define **intermediate shapes** Ω_i of increasing height h_i

$$\Omega_i = \{x \in \Omega \text{ such that } x_d \leq h_i\} \quad 1 \leq i \leq n.$$

- Since the fabrication process operates layer by layer, optimize layer by layer !
- Idea already used in previous works (here, we follow G. Allaire et al., CRAS 2017).
- Minimize the sum of compliances of all intermediate shapes Ω_i.
- Better modeling but higher computational cost



Multi-layer optimization

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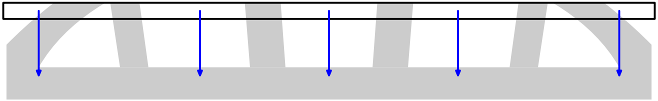

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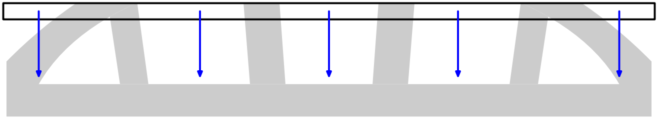

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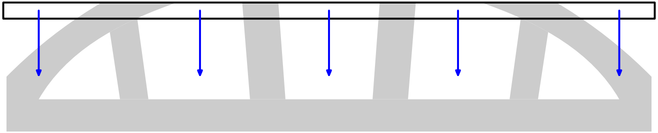

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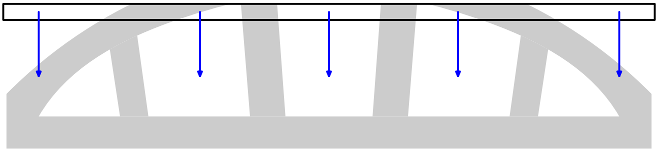

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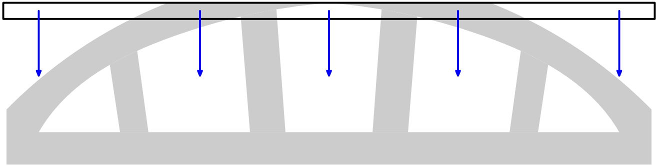

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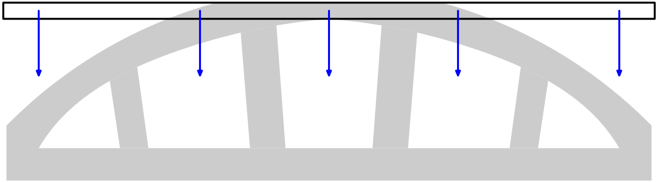

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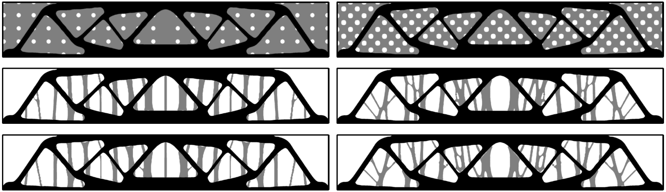
Init.

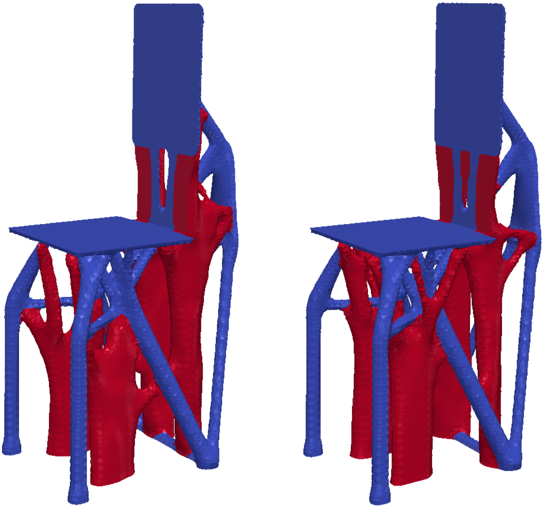
10 Slices

50 Slices



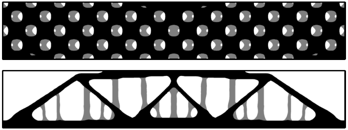
3D chair, layer by layer

5 and 10 slices



- at every iteration we solve **two** state equations : one for the final loads on the structure ω alone and another for the building loads on the supported structure $S \cup \omega$
- evolve the two shapes simultaneously using two level set functions for the parametrization
- different shape derivatives on $\partial\omega \setminus S, \partial S \setminus \omega$ and $\partial\omega \cap \partial S$

The MBB example: video

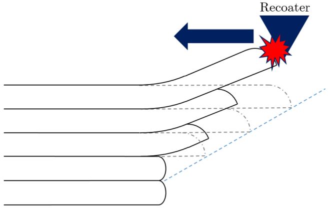


IV - Optimal supports for thermal deformation

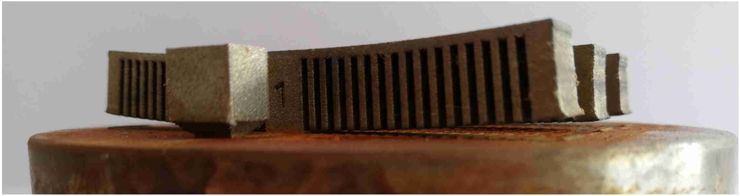
We now optimize supports to minimize thermal deformations or stresses.

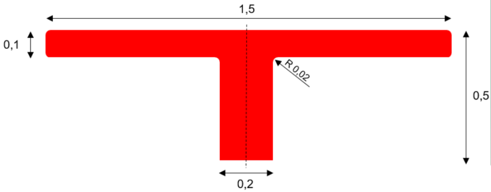
It requires a thermo-mechanical model. For example:

- thermo-elasticity and heat equation (not discussed here),
- inherent strain model.



Sketch of the layer deformation, which can stop the layer deposition, because of thermal retraction upon cooling.





Geometry of T-shape (left), vertical displacement (right) induced by the fabrication process (simulation of a thermo-elastic model).

Inherent strain model

A well-known model for welding process. No heat equation !

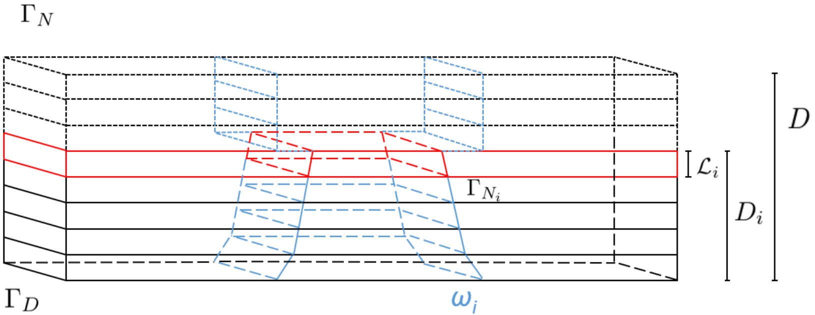
The thermal effects are encoded in a given inherent strain tensor ϵ^*.

Solve the standard quasi-static elasticity equations with a stress tensor defined by

$$\sigma = \sigma^{el} + \sigma^{inh} \quad \text{with} \quad \sigma^{el} = Ae(u) \text{ and } \sigma^{inh} = A\epsilon^*.$$

The inherent strain tensor is calibrated by an inverse problem on a test case. Typically

$$\epsilon^* = \begin{bmatrix} -0.0001 & 0 & 0 \\ 0 & -0.0001 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



Layer by layer construction of the part ω in the build chamber D.

M. Bihr, G. Allaire, X. Betbeder-Lauque, B. Bogosel, F. Bordeu, J. Querois, *Part and supports optimization in metal powder bed additive manufacturing using simplified process simulation*, CMAME 395, 114975 (2022).

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The supported structure $\Omega = \omega \cup S$ is divided into M layers, and each intermediate shape is built from the first i layers such that $\Omega_i = \Omega \cap D_i$. The model is

$$\begin{cases} -\mathrm{div}(\sigma_i) & = & 0 & \text{in } \Omega_i, \\ \sigma_i & = & A\left(e(u_i) + \epsilon^*_{\mathcal{L}_i}\right) & \text{with } \epsilon^*_{\mathcal{L}_i}(x) = \epsilon^* \chi_{\mathcal{L}_i}(x), \\ \sigma_i n & = & 0 & \text{on } \Gamma_{N_i}, \\ u_i & = & 0 & \text{on } \Gamma_D \cap \partial\Omega_i. \end{cases}$$

We consider a criterion

$$J(S) = \sum_{i=1}^{M} \int_{\Omega_i} j(u_i)dx \quad \text{with} \quad j(u_i) = |\max(0, u_i \cdot e_d)|^2 \chi_{\mathcal{L}_i}.$$

The optimization problem is

$$\begin{array}{lccc} \displaystyle\min_{S \subset D \setminus \omega} & J(S) & & \\ \text{such that} & |S| & = & |S_0|, \end{array}$$

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Introduce an adjoint state p_i solution of

$$\begin{cases} -\mathrm{div}(Ae(p_i)) &= -j'(u_i) & \text{in } \Omega_i, \\ (Ae(p_i))n &= 0 & \text{on } \Gamma_{N_i}, \\ p_i &= 0 & \text{on } \Gamma_D. \end{cases}$$

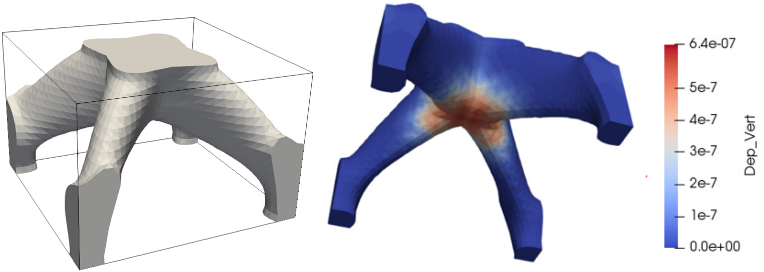
Proposition. The shape derivative in the direction of the vector field $\theta \in W^{1,\infty}(D, \mathbb{R}^d)$ is given by

$$J'(S)(\theta) = \sum_{i=1}^{M} \int_{\partial S \cap D_i} \theta \cdot n \Big(j(u_i) + A \left(e(u_i) + \epsilon^*_{\mathcal{L}_i} \right) : e(p_i) \Big) \, ds.$$

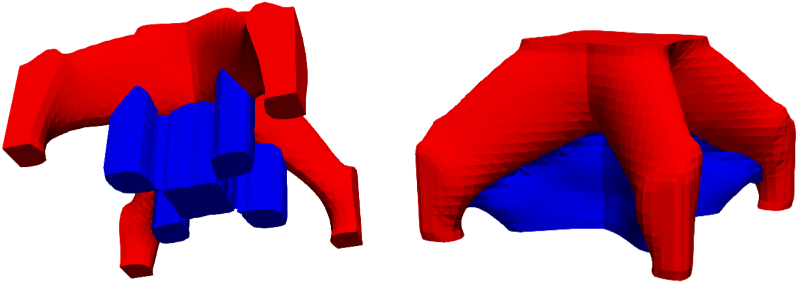
Proof. Introducing the Lagrangian

$$\mathcal{L}(\Omega, \{u_i\}, \{p_i\}) = \sum_{i=1}^{M} \int_{\Omega_i} j(u_i)\, dx + \sum_{i=1}^{M} \int_{\Omega_i} A \left(e(u_i) + \epsilon^*_{\mathcal{L}_i} \right) : e(p_i)\, dx$$

and differentiating $\mathcal{L}$ with respect to all the variables give the desired result.

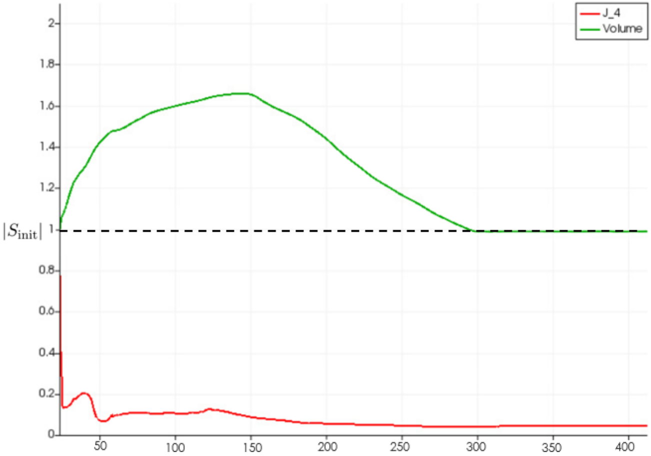


Fixed part ω to build (left) and associated vertical displacements predicted by the inherent strain model (right).



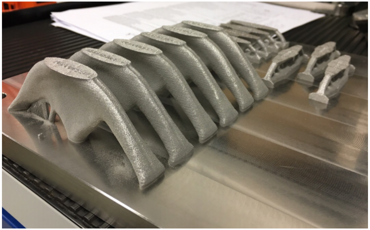
Supports S in blue: initial ones (left) and optimized ones (right) for the fixed part ω in red.

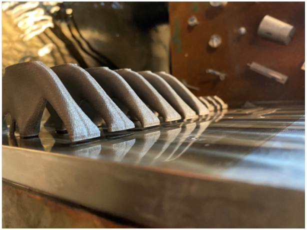
Convergence history

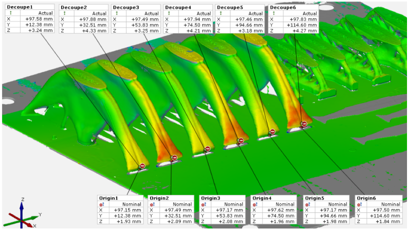


Convergence history for the objective function $J(S)$ (red) and the volume $|S|$ (green).

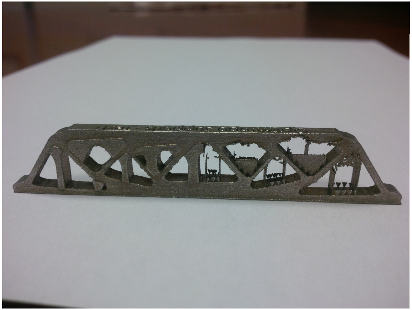
Comparison of deformations for an optimized and a non-optimized structure. Calibration of the inherent strain model.

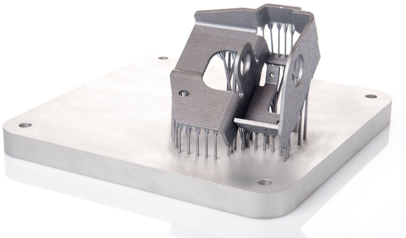






The interface between supports and the built part is made fragile.

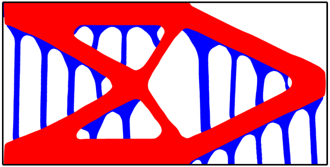




In practice, it is not possible to mesh the fine details at the interface between the built structure and its supports. All the more, for optimization purposes...

Therefore, the tree structure or the dotted line of holes for ease of separation are modeled through an **imperfect interface**.

An interface Γ separates the built structure ω from its support S.



The interface Γ is imperfect, meaning that the displacement is discontinuous through Γ and the normal stress is continuous, proportional to the displacement jump.

Linear elasticity with an imperfect interface

For a smooth applied load F, the displacement u is the solution of

$$\begin{cases} -\mathrm{div}\,\sigma(u) = F & \text{in } \omega \text{ and in } S, \\ u = 0 & \text{on } \Gamma_D, \\ \sigma(u)n = 0 & \text{on } \Gamma_N, \\ [\sigma(u)\nu] = 0 & \text{on } \Gamma = \partial S \cap \partial\omega \\ [u] = -R\sigma(u)\cdot\nu & \text{on } \Gamma = \partial S \cap \partial\omega, \end{cases}$$

with $\sigma(u) = Ae(u) = 2\mu e(u) + \lambda\,\mathrm{div}\,u\,\mathrm{Id}$, $e(u) = \frac{1}{2}(\nabla u + \nabla^t u)$, the jump $[f] = f_\omega - f_S$, $\nu = n_\omega$ the normal to Γ.
The matrix R is the compliance (inverse of rigidity) of the interface

$$R = \alpha(\mathrm{Id} - \nu\otimes\nu) + \beta\nu\otimes\nu,$$

where $\alpha, \beta > 0$ are the tangential and normal compliances.

Denote by u_ω and u_S the restriction of the displacement u in ω and S. Define the broken Sobolev space

$$\mathbf{X}_0 := \left\{ u \in L^2(\Omega)^d : \; u_S \in H^1(S)^d, \; u_\omega \in H^1(\omega)^d, \; u = 0 \text{ on } \Gamma_D \right\}.$$

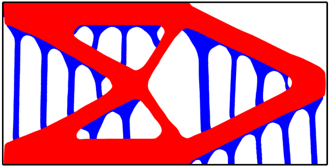
Find $u \in \mathbf{X}_0$ such that, for any $v \in \mathbf{X}_0$

$$\int_\omega \sigma(u_\omega) : e(v_\omega) \, dx + \int_S \sigma(u_S) : e(v_S) \, dx + \int_\Gamma R^{-1}[u] \cdot [v] \, ds = \int_{\omega \cup S} F \cdot v \, dx,$$

where $e(u) = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$ is the *strain tensor* and the *stress tensor* is $\sigma(u) = Ae(u) = 2\mu e(u) + \lambda \, \mathrm{div} \, u \, \mathrm{Id}$.

We consider the system of linearized elasticity for ω and S:

- the built structure ω (in red) is fixed,
- the support S (in blue) is optimized,
- the interface $\Gamma = \partial S \cap \partial\omega$ is imperfect.



Geometrical setting:

- by definition, the interface Γ is constrained to belong to $\partial\omega$.
- the interface Γ is moving tangentially on $\partial\omega$,
- the material parameters between ω and S are often the same.

Optimization problem

The **shape optimization** problem is the compliance minimization

$$\inf_{S \in \mathcal{U}_{ad}} J(S) = \int_{\omega \cup S} F \cdot u \, dx,$$

where the set of admissible supports is typically

$$\mathcal{U}_{ad} = \left\{ S \subset D \setminus \omega \text{ open set such that } \int_S dx = V_0 \right\},$$

where $D \subset \mathbb{R}^d$ is given and V_0 is a prescribed volume.

Reference: G. Allaire, B. Bogosel, M. Godoy, *Topology optimization of supports with imperfect bonding in additive manufacturing*, HAL preprint: hal-03538224 (2022).

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Theorem. Assume $\theta \cdot n = 0$ on $\Gamma = \partial S \cap \partial \omega$. The shape derivative of the compliance is given by

$$J'(S)(\theta) = \int_{\partial S \setminus \partial \omega} \left(-Ae(u) \cdot e(u) + 2F \cdot u \right) \; \theta \cdot n \, ds \\ - \int_{\partial \Gamma} R^{-1}[u] \cdot [u] \theta \cdot \tau \, dl$$

where τ is the tangent vector to $\partial \omega$, normal to Γ, and dl is the $(d-2)$ dimensional measure along $\partial \Gamma$.

Remark. Only the second term is caused by the varying imperfect interface.

Lemma.
Assume that Γ is a smooth surface (of co-dimension 1) in $\mathbb{R}^d$ and $g \in H^2(\mathbb{R}^d)$ is a given function. For any $\theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$ the shape derivative of

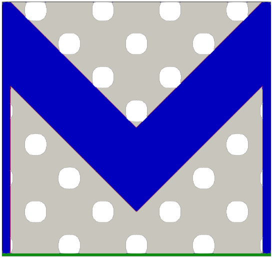
$$J(\Gamma) = \int_\Gamma g \, ds$$

is

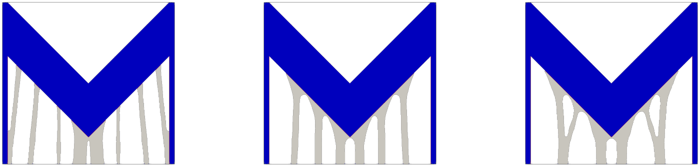
$$\langle J'(\Gamma), \theta \rangle = \int_\Gamma \left(\frac{\partial g}{\partial \nu} + g\kappa \right) \theta \cdot \nu \; ds + \int_{\partial \Gamma} g \, \theta \cdot \tau \; dl,$$

where ν is the unit exterior normal vector to Γ, κ is the mean curvature, τ is the unit tangent vector to Γ such that τ is normal to both $\partial\Gamma$ and ν, and dl is the $(d-2)$ dimensional measure along $\partial\Gamma$.

The structure and supports are fixed on the bottom side. The force is gravity with the same material density.

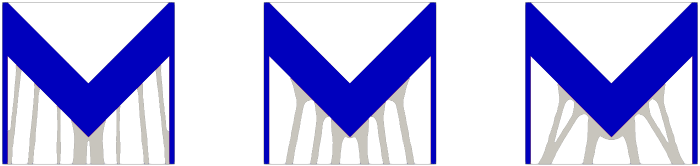


The 'M-part' (light blue) and its support initialization (dark blue). The domain is $D = [-1.6, 1.6]^2$ and $V(\omega) = 3.6$. The objective volume for S is $V_{sup} = 1.0$



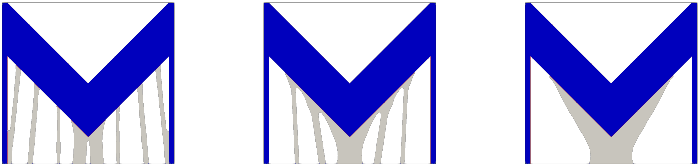
Optimized supports for $\alpha = \beta = 0.001$ (left), $\alpha = \beta = 20$ (center) and $\alpha = \beta = 50$ (right). Perfect interface recovered for small α, β.

Tangentially weak interface



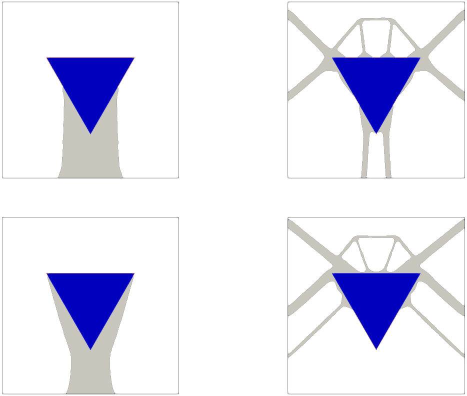
Optimized supports for $\alpha = \beta = 0.001$ (left), $\alpha = 20, \beta = 10$ (center) and $\alpha = 50, \beta = 10$ (right).

Normally weak interface

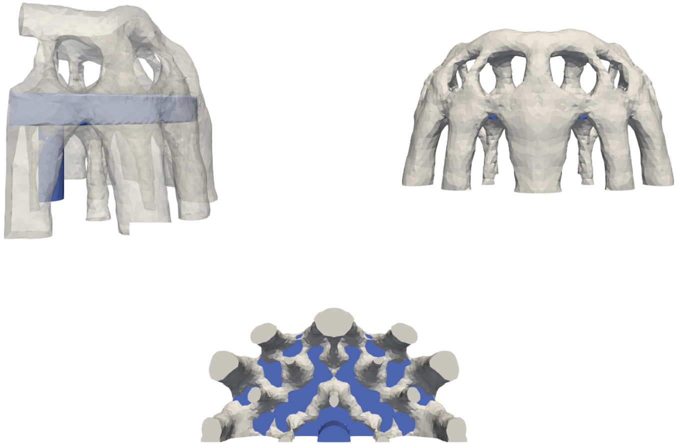


Optimized supports for $\alpha = \beta = 0.001$ (left), $\alpha = 10, \beta = 20$ (center) and $\alpha = 10, \beta = 50$ (right).

Supports can be attached to any side, except the upper one.

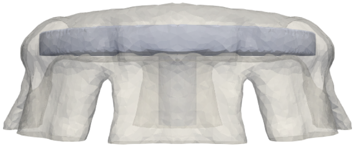


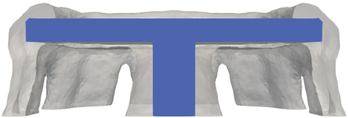
$\alpha = \beta = 0.001$ (upper left), $\alpha = \beta = 50$ (upper right), $\alpha = 10, \beta = 50$ (lower left) and $\alpha = 50, \beta = 10$ (lower right).

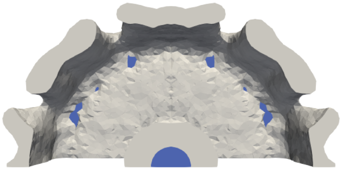


weak interface $\alpha = \beta = 400$

3D table (ctd.)







normally weak interface $\alpha = 1$ and $\beta = 100$

Many opportunities for topology optimization in the context of additive manufacturing !

- Many variants of the objective and of the constraints (B. Bogosel).
- Accessibility of supports for their removal (M. Bihr).
- Lattice materials (P. Geoffroy-Donders, O. Pantz).
- Laser path optimization (M. Boissier, C. Tournier).
- Material anisotropy optimization (A. Touiti, F. Jouve).