Rapid stabilization for the linearized water wave equations and Fredholm backstepping.

Ludovick GAGNON, IECL - INRIA - Nancy
Amaury HAYAT, CERMICS - Ecole des Ponts Paristech - Champs-sur-Marne
Shengquan XIANG, Chaire des EDP - EPFL - Lausanne
Christophe ZHANG, IECL - INRIA - Nancy

Backstepping is one of the most powerful methods to stabilize a system : it consists in finding an isomorphism that transforms the original system, difficult to stabilize, into a target system, easier to stabilize. It is then sufficient to stabilize the target system and to use the inverse mapping to stabilize the original system. The formal question of interest becomes : **Can I find a feedback operator K and an isomorphism T such that the original system is mapped to the target system ?** To simplify the problem, the original approach [3] proposed to look for a Volterra transform of the second kind, which are always invertible. This approach has known a great success in the last 15 years with several hundreds of studies based on it.

However, restricting that much the space of transforms introduce some limitations. For instance, it is hard to deal with a system where the control is internal such as

$$\begin{cases} \partial_t y + \mathcal{A}y + BKy = 0, \\ y(t,\cdot) \in \mathbb{T}, \end{cases} \tag{1}$$

where $\mathcal{A}$ is a differential operator, B is a fixed (potentially unbounded) control operator, K is a feedback operator to be chosen. To tackle these limits, Coron and Lü introduced in 2014 another approach to study the Kuramoto–Sivashinsky equation [1] : it consists in finding T as a general Fredholm transform and using a controllability assumption on the system to show that T is an isomorphism. A central step in this approach is a quadratically close argument which, as a consequence, intrinsically limits the approach to differential operators $\mathcal{A}$ with eigenvalues scaling as n^α where $\alpha > 3/2$.

In this talk we present a new duality-compactness method from [2] that tackles this issue : it allows us to treat the critical case of the water-wave equations, which was an open question since 2017, but also a large class of skew-adjoint differential operators provided that $\alpha > 1$.

[1] J.-M. Coron, Q. Lü. *Fredholm transform and local rapid stabilization for a Kuramoto–Sivashinsky equation.* J. Differential Equations, **259(8)**, 3683–3729, 2015. doi :10.1016/j.jde.2015.05.001.

[2] L. Gagnon, A. Hayat, S. Xiang, C. Zhang. *Fredholm backstepping for critical operators and application to rapid stabilization for the linearized water waves.* arXiv preprint arXiv :2202.08321, 2022.

[3] M. Krstic, A. Smyshlyaev. *Boundary Control of PDEs : A Course on Backstepping Designs*, vol. 16 of *Advances in Design and Control.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2008. doi :10.1137/1.9780898718607.

Contact : `amaury.hayat@enpc.fr`