Stochastic control systems in infinite dimension

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Résumé

We consider :

- $(\Omega, \mathcal{F}, F, P)$ a filtered probability space.
- $(W(t))_{t\geq 0}$ a one dimensional Brownian motion.
- $A : D(A) \subset H \to H$ a generator of a $\mathcal{C}_0$-semigroup $(T(t))_{t\geq 0}$ on H.
- $\mathcal{M} \in \mathcal{L}(H)$ a linear bounded operator.
- $B \in \mathcal{L}(U, H_{-1})$ control operator.

Let $\xi \in L^2_{\mathcal{F}_0}(\Omega, H)$. In the present work, we are concerned with the well-posedness and control properties for distributed stochastic control system

$$\begin{cases} dX(t) = (AX(t) + Bu(t))dt + \mathcal{M}(X(t))dW(t), & t > 0, \\ X(0) = \xi. \end{cases} \tag{1}$$

More precisely, by assuming that the control operator is admissible in deterministic sense and reformulating the studied stochastic differential equation to an abstract stochastic Cauchy problem on a product space, we given a new proof to the existence of a unique mild solution. We further discuss the notion of exact controllability of such systems. An application to bacterial population systems is given.

Théorème 1. *Let (A, B) be admissible. Then there exists a unique mild solution $X(\cdot; \xi, u) \in \mathcal{C}_F(0, +\infty; L^2(\Omega, H))$ of (1) such that*

$$X(t; \xi, u) = T(t)\xi + \int_0^t T(t-s)\mathcal{M}(X(s, \xi, 0))dW(s) + \Phi^W_t u, \qquad t \geq 0,$$

where

$$\Phi^W_t u = \tilde{\Phi}_t u + \int_0^t T(t-s)\mathcal{M}(\Phi^W_s u)dW(s), \quad \textit{and}$$
$$\Phi^W_t \in \mathcal{L}\left(L^2_F(0, +\infty; U), L^2_{\mathcal{F}_t}(\Omega; H)\right), \qquad t \geq 0.$$

See the proof in [1]).

Références

[1] Lahbiri, F. Z., & Hadd, S (2021), A functional analytic approach to infinite dimensional stochastic linear systems, *SIAM Journal on Control and Optimization*, **59**(5), 3762-3786.

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