Numerical optimization of shape and orientation of locally anisotropic materials

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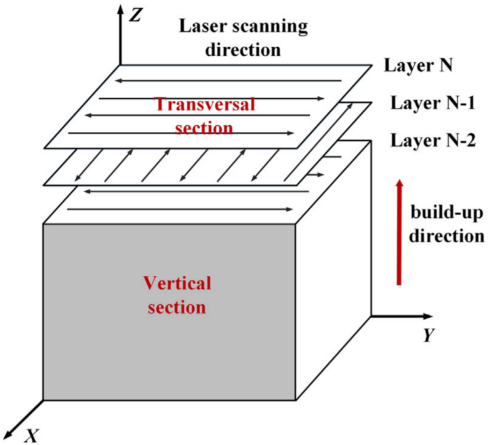
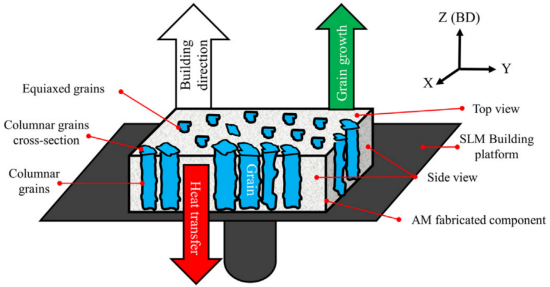
Introduction



Layering and laser scanning in SLM

Z. Chen, & al. 2018, Anisotropy of nickel-based superalloy K418 fabricated by selective laser melting.

Why anisotropy orientation is interesting?



Columnar grains formation in SLM

A. Rezaei, & al. 2020, Microstructural and mechanical anisotropy of selective laser melted IN718 superalloy at room and high temperatures using small punch test

Goals of this work

- Optimize the local anisotropy orientation of 2D and 3D structures and use the **level set method** to optimize its topology:
  - to minimize the compliance of the structure for single and multiloads cases.
  - to achieve a **target displacement**.
- obtain smooth orientation through regularization.
- compare concurrent optimization of orientation and topology to a non concurrent coupled optimization.

Some references on the same problem:

1. *M. Schmidt. 2020. Structural topology optimization with smoothly varying fiber orientations, Structural and Multidisciplinary Optimization*
2. *A. Safonov. 2019. 3D topology optimization of continuous fiber-reinforced structures via natural evolution method, Composite Structures*
3. *G. Allaire, P. Geoffroy-Donders & O. Pantz. 2018, Topology optimization of modulated and oriented periodic microstructures by the homogenization method, Computers & Mathematics with Applications.*

Outline

- Introduction
- I. Topology optimization coupled with explicit optimal orientation method.
  - Problem model
  - Methods
  - Numerical results
- II. Topology optimization coupled with a gradient descent orientation optimization method.
  - Problem model
  - Methods
  - Numerical results
- Summary and conclusions

I. Model: Minimum compliance problem

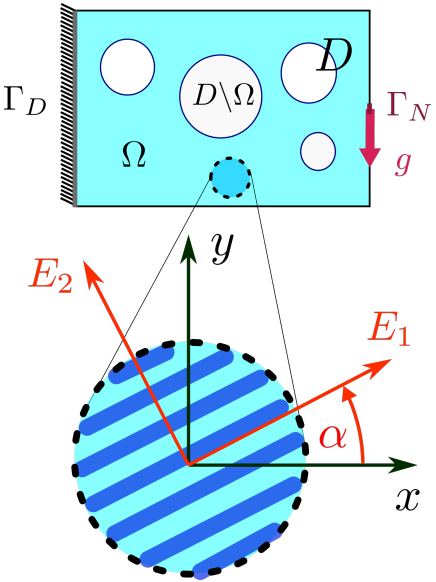
- $\min_{\Omega,\alpha} J(\Omega,\alpha) = \int_{\Gamma_N} g u_{\Omega,\alpha} \mathrm{d}x$
- such that $\int_\Omega dx = V_{\text{tar}}$:

$$\begin{cases} -\operatorname{div}\left(A^*_\Omega(\alpha)\varepsilon\left(u_{\Omega,\alpha}\right)\right) = 0 & \text{in } D \\ u_{\Omega,\alpha} = 0 & \text{on } \Gamma D \\ A^*_\Omega(\alpha)\varepsilon\left(u_{\Omega,\alpha}\right) n = g & \text{on } \Gamma_N \end{cases}$$

- $\varepsilon(u_{\Omega,\alpha}) = \frac{1}{2}\left(\nabla u_{\Omega,\alpha} + (\nabla u_{\Omega,\alpha})^t\right)$
- $\forall x \in D, A^*_\Omega(\alpha)(x) = \begin{cases} A^*(\alpha) & \text{if } x \in \Omega \\ \epsilon A^*(\alpha) & \text{if } x \in D\backslash\Omega \end{cases}$

  ϵ: **ersatz** material density.
- $A^*(\alpha) = R(\alpha) A R(\alpha)^t$

  A: **orthotropic Hooke**'s tensor.



I. Methods: Topology and Shape optimisation

- Suppose α fixed.
- Boundary variation in the direction θ:

$$\Omega_\theta = (I+\theta)(\Omega_0)$$

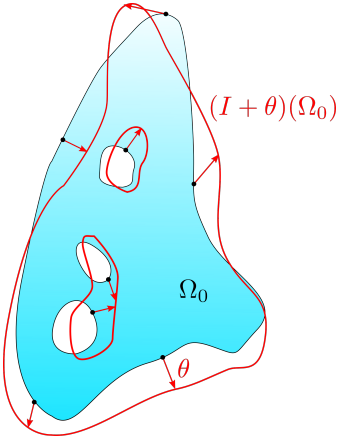
- use Cea's method for shape derivative:

$$\mathcal{L}(\Omega, u, p) = J(u) - \int_\Omega A^*(x)\varepsilon(u)\varepsilon(p)dx + \int_{\Gamma_N} gpds$$

p: adjoint displacement

$$\frac{\mathrm{d}J}{\mathrm{d}\theta}(\theta) = \frac{\partial\mathcal{L}(\Omega,u,p)}{\partial\theta}(\theta) = \int_{\partial\Omega} \underbrace{-A^*(x)\varepsilon(u)\varepsilon(p)}_{V}\theta nds$$

- $\rightarrow$ choose $\theta = -Vn$ with n the normal to the shape.
- define ϕ as a **Level-set map function** for Ω.
- Optimal lagrange multiplier for volume constraint.



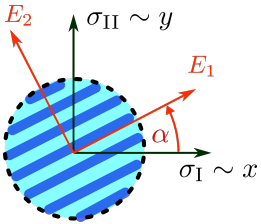
$$\begin{cases} \phi(x) < 0 \text{ if } x \in \Omega \\ \phi(x) = 0 \text{ if } x \in \partial\Omega \\ \phi(x) > 0 \text{ if } x \in D\backslash\bar{\Omega} \end{cases}$$

I. Methods: Optimal orientation for compliance

- suppose the stress field to be constant.
- local minimization problem $\forall x \in D$:

$$\min_\alpha j(\alpha) = (A^*)^{-1}(\alpha)\sigma : \sigma$$

- **Optimality criteria**: $\frac{\mathrm{d}j(\alpha)}{\mathrm{d}\alpha} = 0$

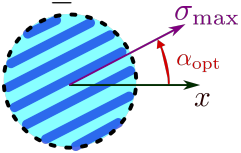


Solution:

- for materials that are **relatively weak** to shear:
  1.direction of the major absolute principal stress.

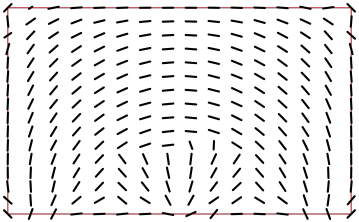
  P. Pedersen. 1990, Bounds on elastic energy in solids of orthotropic materials.

- $E1 \geq E2$

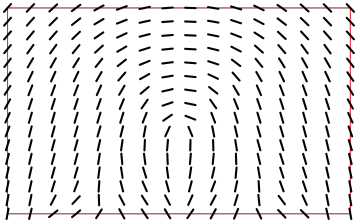


I. Methods: Regularization of the orientation

- $R(\alpha) = \tilde{R}(b)$
- Regularize $b = (\cos(2\alpha), \sin(2\alpha))^t$
- find δb such that $\forall \delta c$:
  $2\int_D A^{*-1}\tilde{R}\,(b+\delta b)\,\sigma : \tilde{R}'(\delta c)\sigma \mathrm{dx} + 2\eta^2 \int_D (b \wedge \nabla\,(b+\delta b)) \cdot (b \wedge \nabla \delta c)\,\mathrm{dx} = 0$



Orientation **before** the regularization



Orientation **after** the regularization

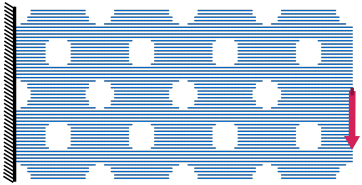
G. Allaire, P. Geoffroy-Donders & O. Pantz. 2018, Topology optimization of modulated and oriented periodic microstructures by the homogenization method.

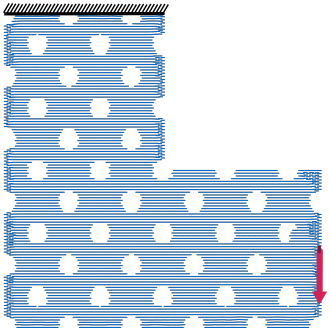
I. Results: Cantilever & Lbeam - Initializations

- **Material Properties**

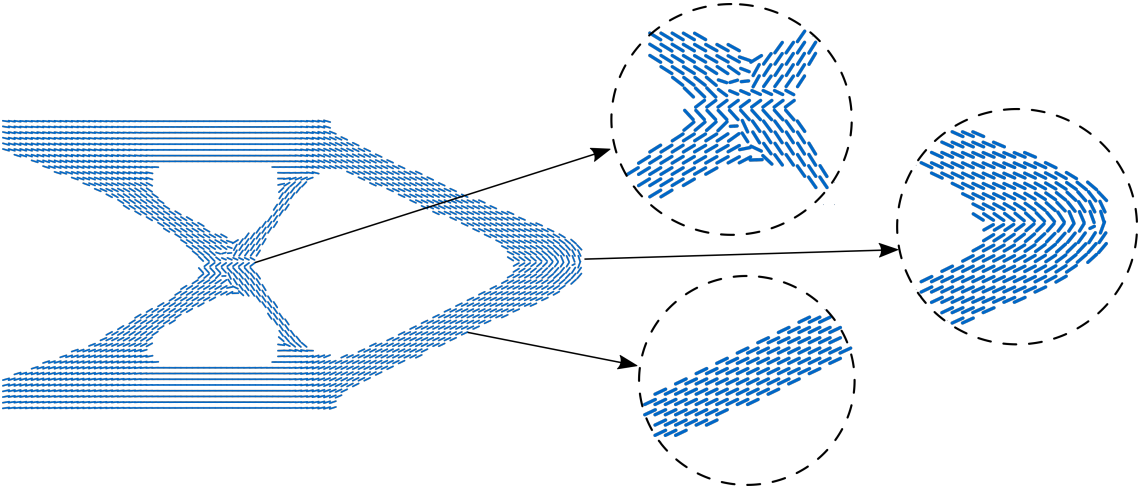
E_1	E_2	G_{12}	ν_{12}	α_0
10	5	2	0.3	0

- applied force: $g = -1$.
- $V_{tar} = 40\%$ of the whole mesh.
- Elements number:
  - Cantilever: 11626 triangle elements
  - Lbeam: 17468 triangle elements:



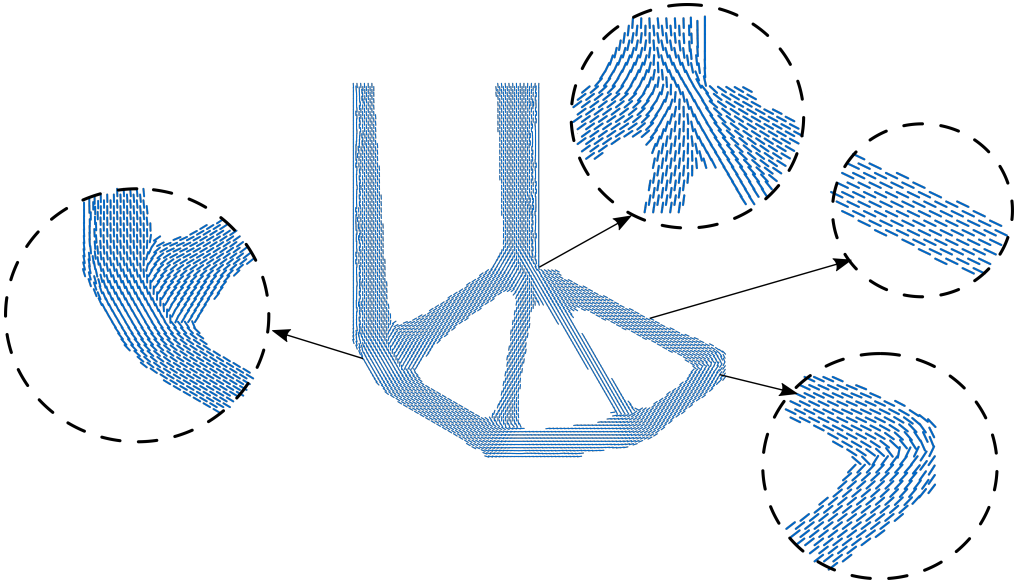


I. Results: Cantilever - Optimized Structure



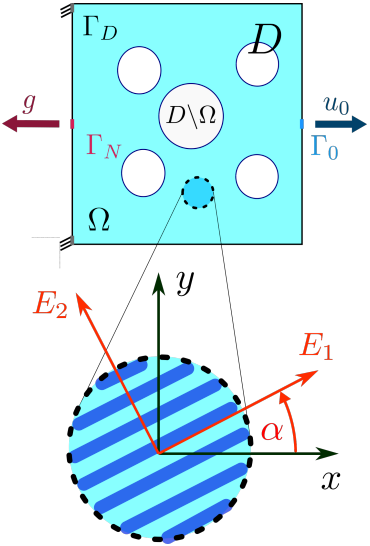
Plot of stiffest direction of the local anisotropy

I. Results: Lbeam - Optimized Structure



II. model: Target displacement u_0 problem

- $\min_{\Omega,\alpha} J(\Omega) = \int_{\Gamma_0} \frac{1}{2}(u_{\Omega,\alpha} - u_0)^2 dx$
- same constraints as the compliance problem.
- $+$ an adjoint problem to solve at each iteration.



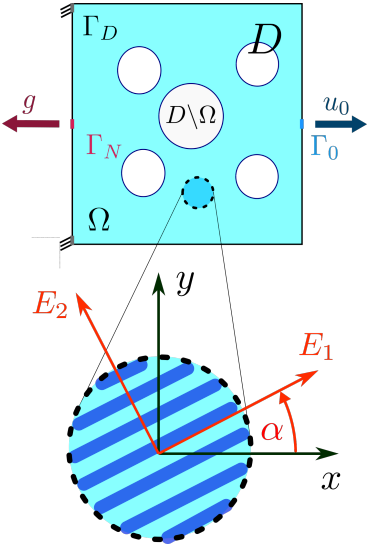
II. Methods: Optimal orientation for Target displacement

- suppose the shape Ω to be fixed
- $\min_\alpha J(\alpha)$ using a **gradient descent** algorithm.
- adjoint method for orientation sensitivity:

$$\mathcal{L}(\alpha, u_\alpha, p_\alpha) = J(u_\alpha) - \int_D A(\alpha)\varepsilon(u_\alpha)\varepsilon(p_\alpha)dx + \int_{\Gamma_N} g p_\alpha ds$$

- $\frac{\mathrm{d}J(\alpha)}{\mathrm{d}\alpha} = \frac{\partial \mathcal{L}(\alpha, u_\alpha, p_\alpha)}{\partial \alpha} = -\frac{\mathrm{d}A^*(\alpha)}{\mathrm{d}\alpha}\varepsilon(u_\alpha) : \varepsilon(p_\alpha)$
- p_α: adjoint displacement, solution of

$$\begin{cases} -\operatorname{div}(A^*(\alpha)\varepsilon(p_\alpha)) = 0 & \text{in } D \\ p_\alpha = 0 & \text{on } \Gamma_D \\ A^*(\alpha)\varepsilon(p_\alpha)\mathbf{n} = u_\alpha - u_0 & \text{on } \Gamma_N \end{cases}$$

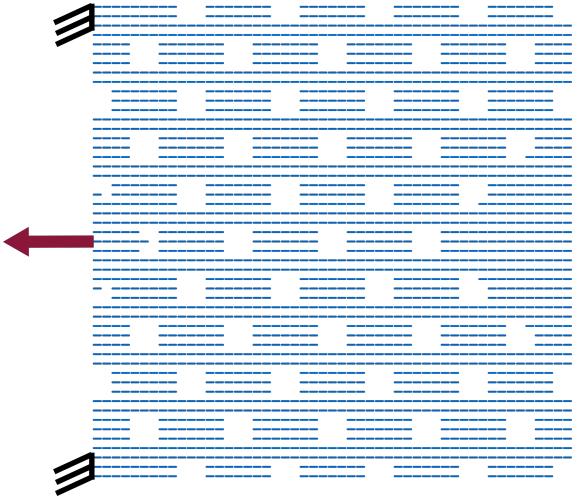


II. Results: Inverter - Initialization

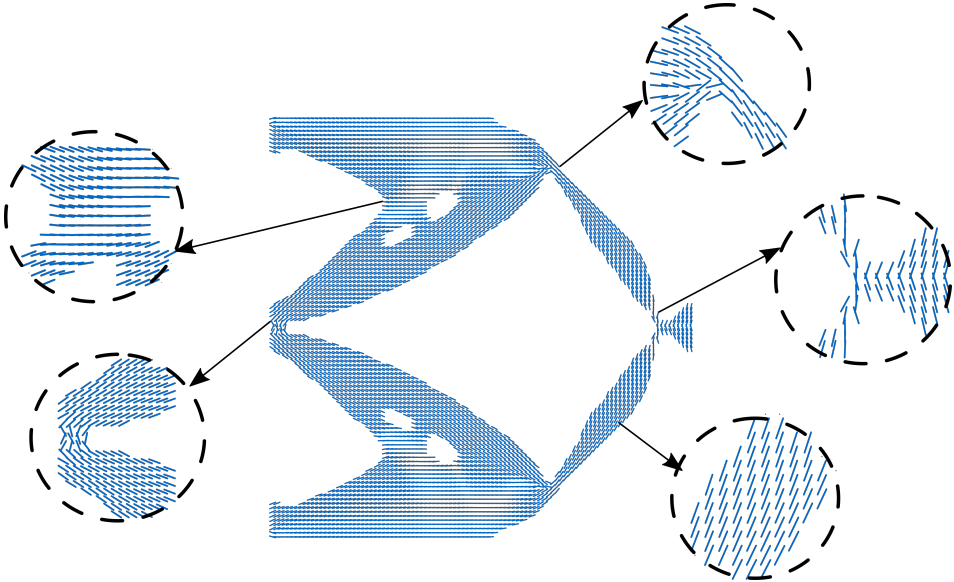
- **Material Properties:**

E_1	E_2	G_{12}	ν_{12}	α_0
10	1	1	0.3	0

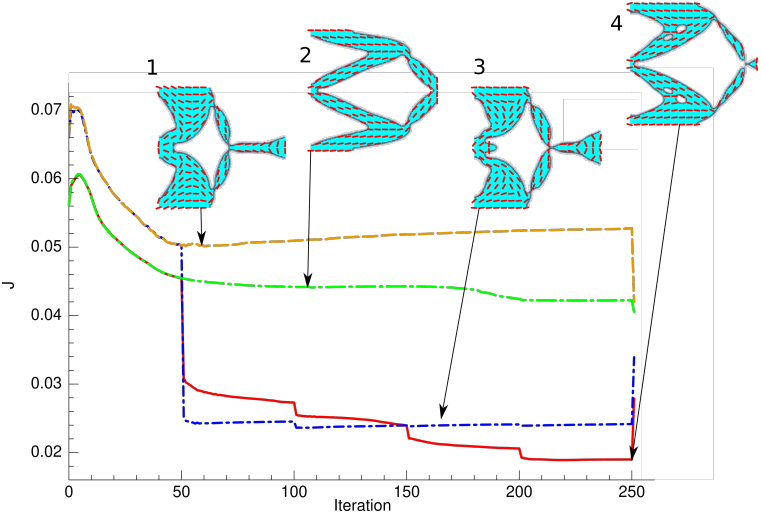
- Applied force: $g = -1$.
- Target displacement: $u_0 = +1$.
- $V_{tar} = 40\%$ of the whole mesh.
- Elements number: 20000 triangle elements
- $u_x = -0.05630157$ at the midpoint of Γ_0



II. Results: Inverter - Optimized structure



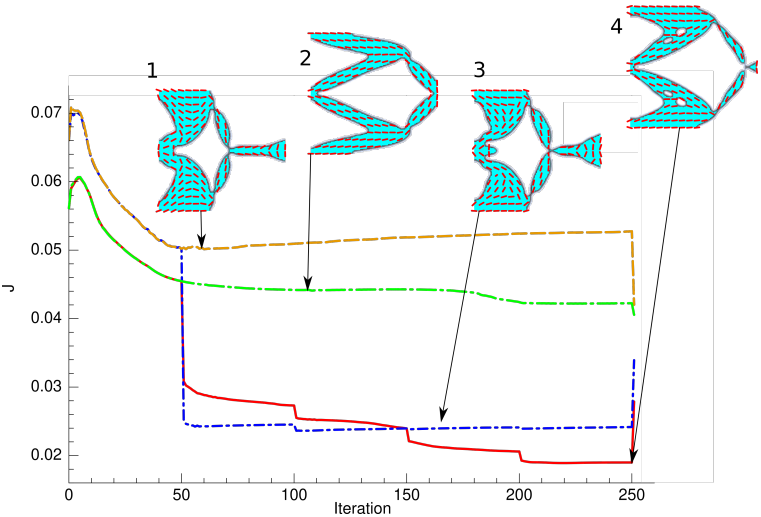
II. Results: Comparison of the inverter results



Evolution of the objective function

1. orientation optimized only in the end with $\alpha_0 = \pi/2$
2. orientation optimized only in the end with $\alpha_0 = 0$
3. concurrent optimization with $\alpha_0 = \pi/2$
4. concurrent optimization with $\alpha_0 = 0$

II. Results: Comparison of the inverter results



Evolution of the objective function

on the midpoint of Γ_0:

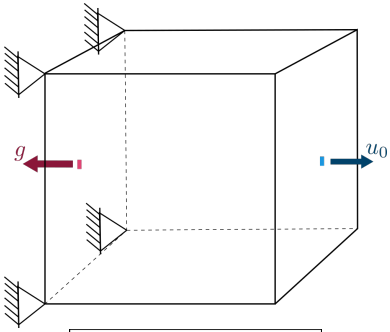
1. $u_x = 0.100107$
2. $u_x = 0.129361$
3. $u_x = 0.208763$
4. $u_x = 0.302197$

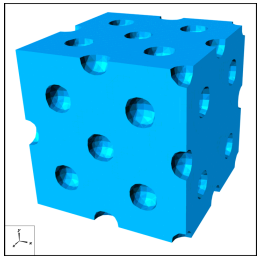
II. Results: Optimal Shape and Orientation in 3D

- **Example:** Displacement inverter
- $V_{tar} = 15\%$ of the whole mesh.
- **Material Properties**
  - transverse isotropic material

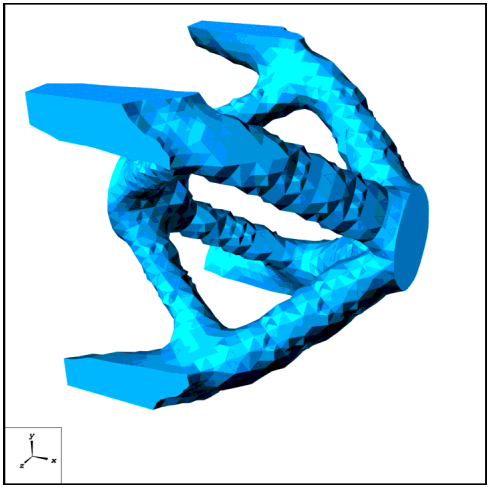
E_n	E_p	G	ν
10	1	1	0.3

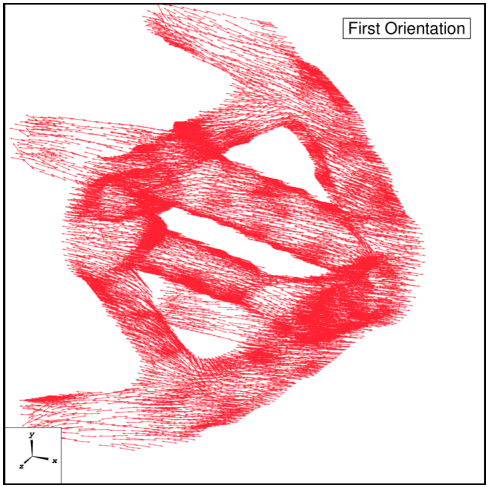
- Node number: ~ 30000
- Initialization with vertical orientation of $E_n \parallel$ to x-axis.





II. Results: Optimal Shape and Orientation in 3D

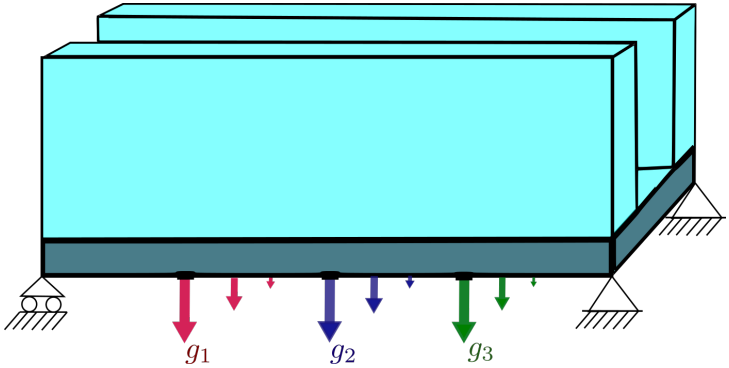




Strategy: optimize shape then orientation

II. model: Multiple load compliance problem

- $\min_{\Omega,\alpha_1,\alpha_2} J(\Omega,\alpha_1,\alpha_2) = \sum_{i=1}^{3} \int_{\Gamma_{N_i}} g_i u_{\Omega,\alpha} \mathrm{d}x$
- same constraints as the compliance problem.

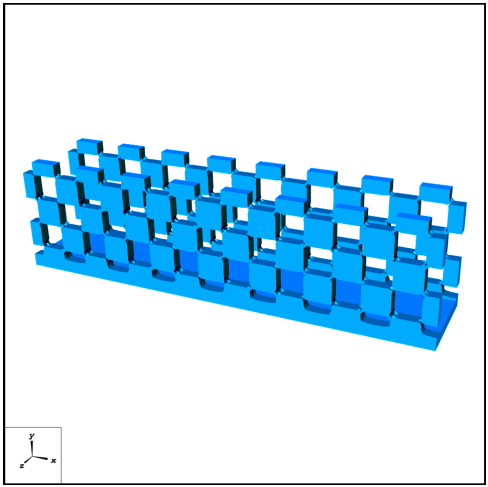


II. Results: Optimal Shape and Orientation in 3D

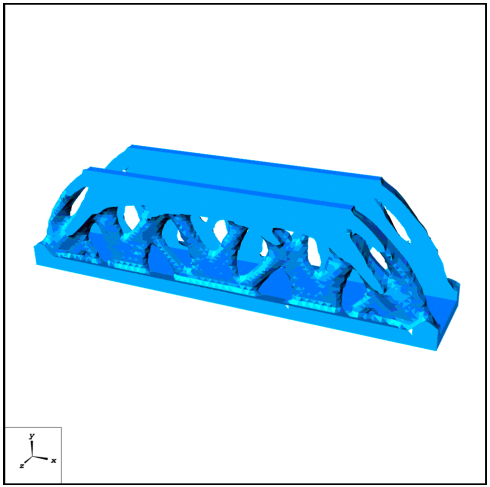
- **Example:** 3D bridge
- $V_{tar} = 30\%$ of the optimizable volume.
- **Material Properties**
  - transverse isotropic material

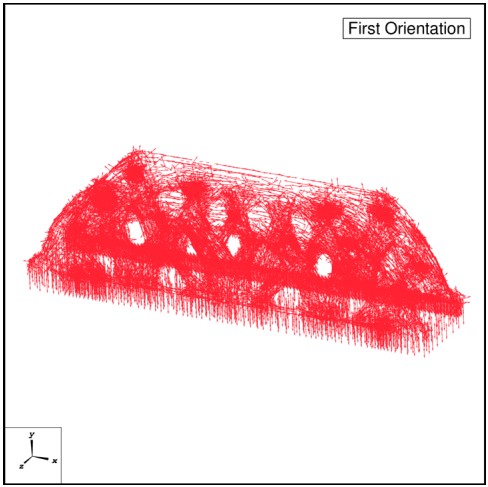
E_n	E_p	G	ν
10	1	1	0.3

- Node number: ~ 40000
- **Strategy**: optimize shape then orientation



II. Results: Optimal Shape and Orientation in 3D

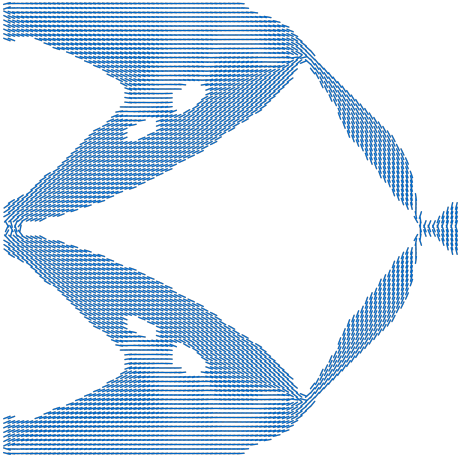




Conclusion

- **Level set** method topology optimization works properly when coupled with the optimization of the **anisotropy** orientation.
- Optimal topology of a structure is heavily **influenced** by the anisotropy of the material.
- In an optimal design that have a local anisotropy, the stiffest direction of the material is **parallel** to the direction of the bars of an optimized structure.
- Optimizing the local orientation is **important** in the case of an anisotropic material.

End.



Thank you for your attention!