Backstepping boundary stabilization of cross-diffusion systems in a one-dimensional moving domain

Jean CAUVIN-VILA, CERMICS, Ecole des Ponts ; Inria MATHERIALS - Paris

Cross-diffusion systems naturally arise in diffusion models of multi-species mixtures [3] and in particular in materials science to model atomic diffusion within solids. Indeed, hydrodynamic limits of some stochastic lattice hopping models [4] read as cross-diffusion systems of the form :

$$\partial_t u - \operatorname{div}_x (A(u)\nabla_x u) = 0, \tag{1}$$

where $u := (u_1, \dots, u_n)$ models the local volumic fractions ($\sum_j u_j \leq 1,\ u_i \geq 0\ \forall i$) of the different species and $A : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the diffusion matrix. In [1] the authors considered a cross-diffusion system defined in a one-dimensional moving boundary domain in order to model a Physical Vapor Deposition process (PVD) used for the fabrication of thin film layers in the photovoltaic industry. The process can be described as follows : a wafer is introduced in a hot chamber where chemical elements are injected under gaseous form. As the latter deposit on the substrate, a heterogeneous solid layer grows upon it. Because of the high temperature conditions, diffusion occurs in the bulk until the wafer is taken out and the system is freezed.

Let $\phi(t) = (\phi_1(t) \geq 0, \dots, \phi_n(t) \geq 0)$ represent the fluxes of the different species absorbed at the surface of the film layer. Then the thickness $e(t)$ of the film evolves according to :

$$e(t) = e_0 + \int_0^t \sum_{i=1}^n \phi_i(s)ds, \tag{2}$$

and the absorption condition on the boundary reads :

$$(A(u)\partial_x u)(t, e(t)) + e'(t)u(t, e(t)) = \phi(t). \tag{3}$$

This work is concerned with the stabilization of (1)-(2)-(3) around uniform equilibrium states using the control variables $(\phi_i)_{1\leq i\leq n}$ in (3). Under appropriate assumptions on A related to the *entropic structure* [3], we show rapid and even finite-time stabilization in L^2 of the *linearized* system. The feedback control is derived using the backstepping technique inspired from [2], which consists in transforming the original system, hard to stabilize, into a simpler target system, using an isomorphism. The difficulties come from the time-dependence of the system that complicates the stability analysis and from the boundary condition (3) that leads to an only weakly defined feedback control in the L^2 setting.

Upcoming work will be concerned with the consequences in terms of *local* stabilization of the *nonlinear* system. This is joint work with Virginie Ehrlacher and Amaury Hayat at Ecole des Ponts.

[1] A. Bakhta, V. Ehrlacher. *Cross-diffusion systems with non-zero flux and moving boundary conditions.* ESAIM : Mathematical Modelling and Numerical Analysis, **52(4)**, 1385–1415, 2018. doi : 10.1051/m2an/2017053.

[2] J.-M. Coron, H.-M. Nguyen. *Null Controllability and Finite Time Stabilization for the Heat Equations with Variable Coefficients in Space in One Dimension via Backstepping Approach.* Archive for Rational Mechanics and Analysis, **225(3)**, 993–1023, 2017. doi :10.1007/s00205-017-1119-y.

[3] A. Jüngel. *Entropy Methods for Diffusive Partial Differential Equations.* SpringerBriefs in Mathematics. Springer International Publishing, Cham, 2016. doi :10.1007/978-3-319-34219-1.

[4] J. Quastel. *Diffusion of color in the simple exclusion process.* Communications on Pure and Applied Mathematics, **45(6)**, 623–679, 1992. doi :10.1002/cpa.3160450602.

Contact : `jean.cauvin-vila@enpc.fr`