# University-industry collaboration : Modeling and simulation for gas distribution.

Nora Aïssiouene, Marguerite D'Olce, Pierre-Yves Lagrée, Giuseppe Parasiliti Rantone, Yohan Penel

CANUM 2022 - Evian-Les-Bains

13 - 17 juin 2022

# 1 - INTRODUCTION

## 1.1 - SUMMIT (SORBONNE UNIVERSITÉ MAISON DES MODÉLISATIONS, INGÉNIERIES ET TECHNOLOGIES)

- Our university: SORBONNE UNIVERSITÉ
- Our services: provide high-quality engineering research to private and public partners.
- Our expertise:
  - Modeling Simulation and Optimization.
  - Data science, IA.
  - Hybrid modeling.
- Some projects:
  - Obepine - Genome quantification in wastewaters / development of smoother / model reduction of compartmental models etc.
  - Eau de Paris - anomaly detection / classification / calibration etc.
  - **GTT - Low Mach model for gas transport.**

# 1 - INTRODUCTION

## 1.2 - History of a collaboration for gas transport with GTT.

# 1 - INTRODUCTION

## 1.3 - Issue

1. Complete compressible Navier-Stokes Equation:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathrm{u}) &= 0 \\
\frac{\partial \rho \mathrm{u}}{\partial t} + \mathrm{div}(\rho \mathrm{u} \otimes \mathrm{u}) + \nabla P &= \rho \mathrm{g} + \mathrm{div}(\sigma) \\
+ \text{ State Equation} & \\
+ \text{ Temperature Equation} &
\end{aligned}
$$

$(\mathrm{u} \otimes \mathrm{w} = \mathrm{u}\mathrm{w}^T)$

2. Compressible: good representation of the physical phenomena but not applicable in practice ( CFL condition too restrictive in the low Mach number regime)
3. Incompressible: not enough to obtain a good representation
4. **Proposition: build a quasi-incompressible model**:
   - Good physical representation + Applicable numerical scheme

# 2 - Objective

## 2.1 - A quasi-incompressible model

- Obtain a model of the form:

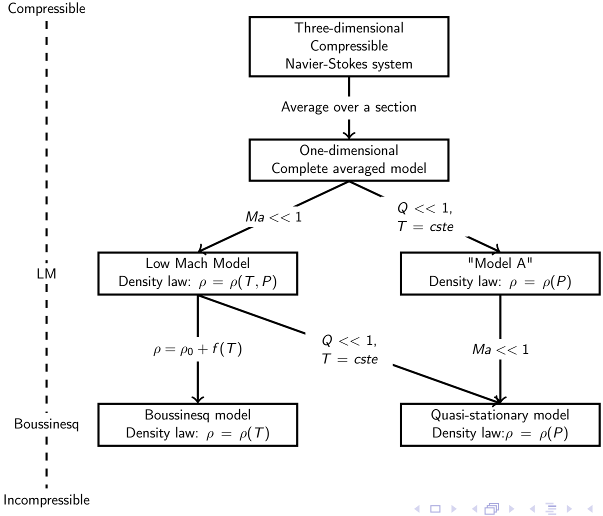
$$
\begin{aligned}
\mathrm{div}(\boldsymbol{u}) &= A(T,P)P'(t) + B(T,P)\mathrm{div}(q_T), && (1)\\
\rho D_t \mathrm{u} &= -\nabla\Pi + \frac{1}{Re}\mathrm{div}(\boldsymbol{\sigma}) - \frac{\rho}{Fr}, && (2)\\
C_p D_t T &= (-\frac{T}{\rho}\left.\frac{\partial\rho}{\partial T}\right|_P)P'(t) - \frac{1}{Re\,Pr}\mathrm{div}(q_T), && (3)\\
\rho &= \rho(T,P), && (4)
\end{aligned}
$$

with

- $\Pi$: the dynamic pressure
- $P$: the thermodynamic pressure
- Equation (1) : the quasi-incompressibility constraint
- Equation (4) : equation of state

# 2 - Objective

## 2.1 - A quasi-incompressible model



# 3 - Modeling

## 3.1 - $1D$ Averaged Navier-Stokes equations for ideal gases

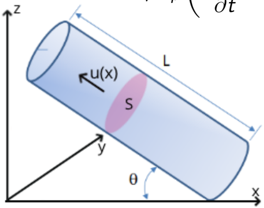
$1D$ AVERAGED NAVIER-STOKES EQUATIONS
for ideal gases

$$\frac{\partial S\rho}{\partial t} + \frac{\partial S\rho u}{\partial x} = 0,$$

$$\frac{\partial S\rho u}{\partial t} + \frac{\partial S\rho u^2}{\partial x} + \frac{\partial SP}{\partial x} = -\tau\pi D - \rho S g \sin\theta,$$

$$\rho C_p \left(\frac{\partial ST}{\partial t} + u\frac{\partial ST}{\partial x}\right) = D_t(SP) + u\tau\pi D - 2\pi R q_w,$$

$$\rho = \frac{P}{rT}.$$



# 3 - Modeling

## 3.2 - Derivation

- Characteristic values $L_c, U_c, t_c$ such that $L_c = U_c t_c$
- Dimensionless variables $\tilde{x} = \frac{x}{L_c}, \tilde{t} = \frac{t}{t_c}, \tilde{u} = \frac{u}{U_c}, \tilde{\rho} = \frac{\rho}{\rho_c}$

$$\frac{\partial \tilde{S}\tilde{\rho}}{\partial \tilde{t}} + \frac{\partial \tilde{S}\tilde{\rho}\tilde{u}}{\partial \tilde{x}} = 0,$$

$$\frac{\partial \tilde{S}\tilde{\rho}\tilde{u}}{\partial \tilde{t}} + \frac{\partial \tilde{S}\tilde{\rho}\tilde{u}^2}{\partial \tilde{x}} + \frac{1}{\gamma Ma^2}\frac{\partial \tilde{S}\tilde{P}}{\partial \tilde{x}} = -\frac{f}{2}\pi\tilde{\rho}\tilde{u}^2\tilde{D} - \frac{1}{Fr}\tilde{S}\tilde{\rho}g\sin\theta,$$

$$\tilde{C}_p\tilde{\rho}\left(\frac{\partial \tilde{S}\tilde{T}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{S}\tilde{T}}{\partial \tilde{x}}\right) = \frac{\gamma-1}{\gamma}D_{\tilde{t}}(\tilde{S}\tilde{P}) + (\gamma-1)Ma^2\frac{f}{2}\pi\tilde{\rho}\tilde{u}^3\tilde{D} - \frac{2\pi\tilde{R}}{Pr\,Re}\tilde{q}_w.$$

$$\boxed{Ma^2 = \frac{\rho_c u_c^2}{\gamma P_c}} \quad \boxed{Fr = \frac{u_c^2}{L_c g}} \quad \boxed{Re = \frac{u_c L_c \rho_c}{\mu_c}} \quad \boxed{Pr = \frac{\mu_c C_{pc}}{k_c}}$$

# 3 - Modeling

## 3.2 - Derivation

Assyptotic developement:

$$
\begin{aligned}
\tilde{u}(x,t) &= \tilde{u}_0(x,t) + Ma\,\tilde{u}_1(x,t) + \mathcal{O}(Ma^2),\\
\tilde{P}(x,t) &= \tilde{P}_0(x,t) + Ma\,\tilde{P}_1(x,t) + Ma^2\Pi(x,t) + \mathcal{O}(Ma^3).
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \tilde{S}\tilde{\rho}_0}{\partial \tilde{t}} + \frac{\partial \tilde{S}\tilde{\rho}_0\tilde{u}_0}{\partial \tilde{x}} &= 0,\\
\frac{\partial \tilde{S}\tilde{\rho}_0\tilde{u}_0}{\partial \tilde{t}} + \frac{\partial \tilde{S}\tilde{\rho}_0\tilde{u}_0^2}{\partial \tilde{x}} + \frac{1}{\gamma}\frac{\partial \tilde{S}\Pi}{\partial \tilde{x}} &= -\frac{f}{2}\pi\tilde{\rho}_0\tilde{u}_0^2\tilde{D} - \frac{1}{Fr}\tilde{\rho}_0\tilde{S}g\sin\theta,\\
\tilde{\rho}_0\tilde{C}_p\left(\frac{\partial \tilde{S}\tilde{T}_0}{\partial \tilde{t}} + \tilde{u}_0\frac{\partial \tilde{S}\tilde{T}_0}{\partial \tilde{x}}\right) &= \frac{\gamma-1}{\gamma}\tilde{S}\tilde{P'}_0 - \frac{2\pi\tilde{R}}{PrRe}\tilde{q}_w.
\end{aligned}
$$

# 3 - Modeling

## 3.2 - Derivation

$$\frac{\partial Su}{\partial x} = -\frac{S}{\gamma P} P'(t) - \frac{2\pi R(\gamma - 1)}{\gamma P} q_w, \quad (6)$$

$$\partial_t u + u \partial_x u + \frac{\partial_x \Pi}{\rho} = -\frac{f}{2} \pi u^2 \frac{D}{S} - g \sin\theta, \quad (7)$$

$$\rho C_p \left( \frac{\partial ST}{\partial t} + u \frac{\partial ST}{\partial x} \right) = S P'(t) - 2\pi R q_w. \quad (8)$$

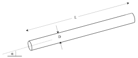
We will denote

$$\eta = \frac{\partial Su}{\partial x}$$

# 3 - Modeling

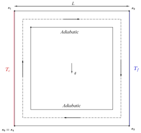
## 3.3 - Boundary conditions

$P$ EQUATION



OPEN DOMAINS

CLOSED DOMAINS



$\boxed{P \text{ GIVEN}}$

$$\boxed{P'(t) = -\frac{2\pi R(\gamma - 1)}{S|\Omega|} \int_{\Omega} q_w}$$

# 4 - Numerical Method

## 4.1 - Equation for $T$

Method of characteristics

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) = f(x,t) = \frac{(\gamma-1)T(x,t)}{\gamma P(t)}P'(t) - \frac{2\pi R(\gamma-1)T(x,t)}{S\gamma P(t)}q_w$$

$$\downarrow$$

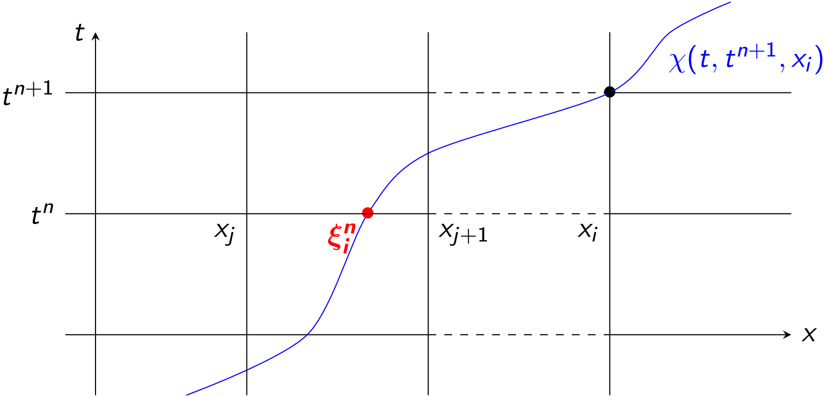
$$T_i^{n+1} = \hat{T}_i^n + \Delta t f(T_i^{n+1}, P^n, \rho^n)$$

$$\downarrow$$

$$\boxed{T_i^{n+1} = \frac{\hat{T}_i^n + \Delta t\left(\frac{(\gamma-1)T_i^n}{\gamma P(t^n)}P'(t^n) + \frac{2\pi R(\gamma-1)T_i^n h}{\gamma S P(t^n)}T_{ref}\right)}{1 + \Delta t\frac{2\pi R(\gamma-1)T_i^n h}{\gamma S P(t^n)}}}$$

# 4 - Numerical Method

## 4.2 - Method of characteristics



# 4 - Numerical Method

## 4.3 - Equation for Π

Equation for Π and $u$:

- Elliptic equation for Π:

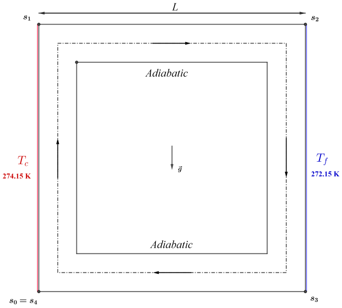
$$-\partial_x \left( \frac{\partial_x \Pi}{\rho} \right) = \partial_t \eta + u \partial_x \eta + \eta^2 + f \pi u \eta \frac{D}{S} - g\delta(x - nL).$$

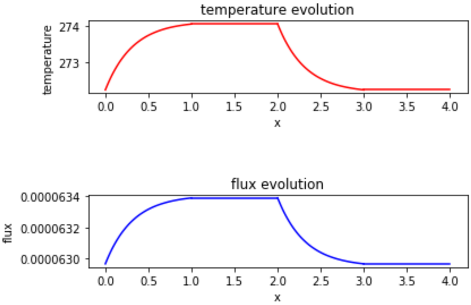
- Method of characteristics for $u$:

$$u_i^{n+1} = \frac{\hat{u}_i^n + \Delta t \left( -\frac{rT_i^{n+1}}{P(t^{n+1})} \frac{\Pi_i^{n+1} - \Pi_{i-1}^{n+1}}{\Delta x} - g sin\theta \right)}{1 + \Delta t \frac{f}{S} \pi D}$$

# 5 - Pseudo analytical solution

## 5.1 - Periodic case



# 5 - Pseudo analytical solution

## 5.1 - Periodic case

$$
\begin{aligned}
\frac{\partial Su}{\partial x} &= -\frac{S}{\gamma P}P'(t) - \frac{2\pi R(\gamma-1)}{\gamma P}q_w, \\
\partial_t u + u\partial_x u + \frac{\partial_x \Pi}{\rho} &= -\frac{f}{2}\pi u^2 \frac{D}{S} - g\sin\theta, \\
\rho C_p\left(\frac{\partial ST}{\partial t} + u\frac{\partial ST}{\partial x}\right) &= SP'(t) - 2\pi R q_w.
\end{aligned}
$$

# 5 - Pseudo analytical solution

## 5.1 - Periodic case

$$T(x) = T_{ref} + (T_0 - T_{ref})e^{-\frac{2\pi R(\gamma-1)h}{\gamma P\Gamma}x}.$$

$$Q(x) = \Gamma T(x).$$

$$\Pi(x) = \frac{bP\Gamma}{rBhT_{ref}}(\ln T(x) - \ln T_0) + \frac{P}{rT_{ref}}(aT_{ref}\Gamma - b)x + \Pi_0.$$

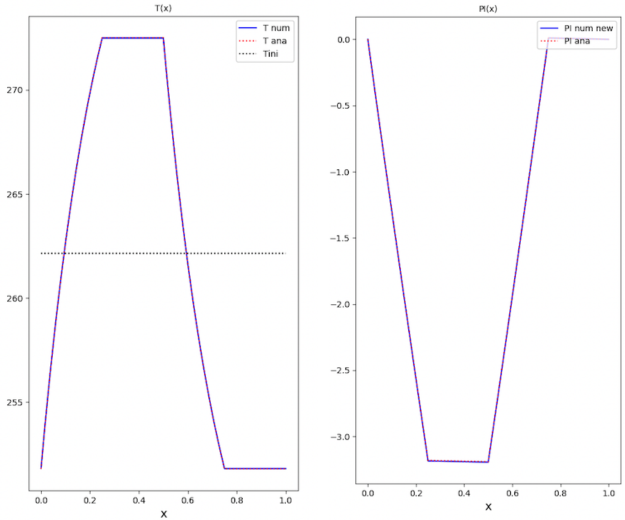
Where:

$$\Gamma := \frac{Q_0}{T_0} = \frac{Q_1}{T_1}.$$

$$f(\Gamma) = 0.$$

# 5 - Pseudo analytical solution

## 5.2 - Numerical results - analytical solution

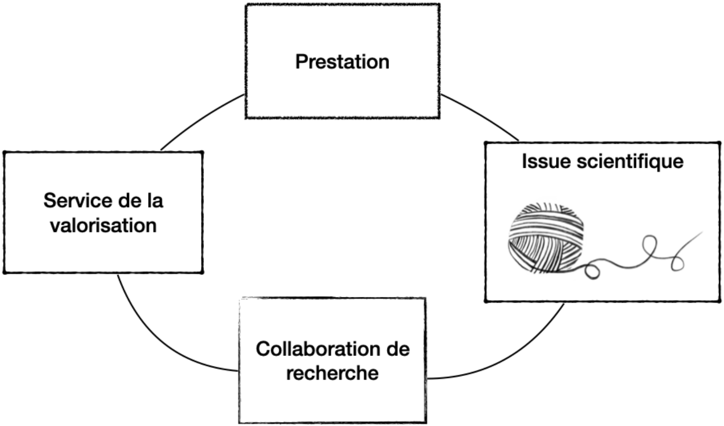


# 6 - Conclusion

- We have developed a one dimensional low Mach model.
- We provided a set of analytical solutions.
- We have given a validation of the thermosiphon solution.

# 6 - Conclusion

## 6.1 - A long-term research collaboration



Thank you !

nora.aissiouene@sorbonne-universite.fr

[1] B. De Andrés-Toro A. Herrán-González, J. De La Cruz and J. RiscoMartín.
Modeling and simulation of a gas distribution pipeline network. 33:1584–1600, 2009.

[2] Carnot smiles.
Part *I*: Development of a low mach number model for a gas distribution pipeline network. 2

[3] Thierry Gallouet, Gary Robert Eymard and Raphaèle Herbin.
Finite volume methods. October 2006.

[4] F. Boyer. Méthodes de volumes finis pour les écoulements en milieux poreux.
Laboratoire d'Analyse, Topologie et Probabilités CNRS.

[5] Y. Penel.
Theoretical and numerical study of a moving interface between two immiscible fluids at a lo
number. PhD thesis, Université Paris-Nord -Paris XIII, Dec.2010.