A class of conservative L^2-stable schemes for the compressible fluid models on staggered grids

Katia AIT AMEUR, Ecole Polytechnique - Palaiseau
Michael NDJINGA, Université de Paris-Saclay - CEA Saclay

Finite volume schemes on staggered grids are popular among thermalhydraulics engineers for their correct low Mach number asymptotic expansion and the absence of checkerboard type spurious oscillations [5]. However, they are generally non conservative, and their stability analysis has historically been performed with a heuristic approach and the tuning of numerical parameters ([4]).
In this talk, we first investigate the linear L^2-stability of staggered schemes by analysing their numerical diffusion operator. For most classical staggered schemes ([5, 1, 2, 3]), our analysis shows that the numerical diffusion is highly nonlinear and we are able to prove the linear stability only in the case of constant sign velocities.
We then propose a new class of conservative linearly L^2-stable staggered schemes for compressible fluid models on staggered grids. The schemes are based on a carefully chosen numerical diffusion operator and the proof of stability follows from the symmetrisation of the system. An important remark is that unlike Godunov type schemes on colocated grids, the numerical diffusion operator of a symmetric system is not symmetric. This property is fundamental to ensure precision and avoid spurious checkerboard modes oscillations for low Mach number flows. Indeed, a Total Variation analysis shows that it is the skew-symmetric part of the diffusion matrix that allows the control of the spurious oscillations. We give examples of such new conservative staggered schemes and present some numerical results showing the good behaviour of the method for 1D and 2D multiphase flows at both low (vortex flow) and high Mach (shock tube) numbers.

[1] R. Herbin, W. Kheriji, J.-C. Latché. *Staggered schemes for all speed flows.* ESAIM : Proceedings, EDP Sciences, Congrès National de Mathématiques Appliquées et Industrielles, 35, pp.122-150, 2011.

[2] R. Herbin, W. Kheriji, J.-C. Latché. *On some implicit and semi-implicit staggered schemes for the shallow water and euler equations.* ESAIM : Mathematical Modelling and Numerical Analysis, EDP Sciences, 48 (6), pp.1807-1857, 2014.

[3] R. Herbin, J.-C. Latché. *A kinetic energy preserving convection operator for the mac discretization of compressible navier-stokes equations.* Mathematical Modelling and Numerical Analysis, 2010. `https://hal.archives-ouvertes.fr/hal-00477079/document`.

[4] C. W. Hirt. *Heuristic stability theory for finite difference equations.* J. Comp. Phys., 2, pp. 339-355, 1968.

[5] A. Prosperetti, G. Tryggvason. *Computational methods for multiphase flow.* Cambridge University press, 2009.

Contact : `aitameur.katia@gmail.com`