Time parallelization, observers and data assimilation

Julien SALOMON, INRIA Paris - Paris **Lucas PERRIN**, INRIA Paris - Paris

Assimilation and identification problems related to hyperbolic systems arise in many fields of applications, e.g. weather forecasting or seismology [1, 5, 6]. Despite the growing importance of computational issues in these fields, to the best of our knowledge, time parallelization of the assimilation procedures has never been investigated either from a practical or from a mathematical point of view. On the other hand, the use of such parallelization techniques for optimal control problems is now well documented. The processing of data arriving as a continuous stream adds a new level of difficulty, both for the assimilation method, which can no longer be based on adjoint computation, and for time parallelization, which usually applies to simulations on bounded, predefined time intervals. The problem of adjoint-free assimilation is usually dealt with by observers, also called nudging techniques [2]. Adapting parallelization techniques in time is the core of this presentation.

Our aim is to present a coupling between a time parallelization method and an observer, in order to accelerate the data assimilation procedure over unbounded time intervals. We will focus on the algorithm *ParaExp* [3] for the first part, and the *Luenberger observer* [4] for the second one. We will present both problems individually, and then our solution for applying the *ParaExp* algorithm onto the *Luenberger observer* over and unbounded time interval. We will then illustrate the performance of this technique with some numerical examples over systems governed by evolution partial differential equations (PDEs), specifically parabolic and hyperbolic problems.

[1] M. Asch, M. Bocquet, M. Nodet. *Data assimilation : methods, algorithms, and applications.* Fundamentals of Algorithms. SIAM, 2016.

[2] D. Auroux, J. Blum, G. Ruggiero. *Data assimilation for geophysical fluids : the Diffusive Back and Forth Nudging*, vol. 15 of *INdAM Series*, pp. 139–174. Springer, 2016.

[3] M. J. Gander, S. Güttel. *Paraexp : A parallel integrator for linear initial-value problems.* SIAM Journal on Scientific Computing, **35(2)**, C123–C142, 2013. doi :10.1137/110856137.

[4] D. Luenberger. *An introduction to observers.* Automatic Control, IEEE Transactions on, **16**, 596 – 602, 1972. doi :10.1109/TAC.1971.1099826.

[5] D. Simon. *Optimal State Estimation : Kalman, H Infinity, and Nonlinear Approaches.* Wiley-Interscience, 2006.

[6] C. K. Wikle. *Atmospheric modeling, data assimilation, and predictability.* Technometrics, **47(4)**, 521–521, 2005. doi :10.1198/tech.2005.s326.

Contact : `lucas.perrin@inria.fr`