# Propagation of acoustic and gravity waves in the ocean: a new derivation for a general model

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Several authors have proposed to use the propagation of acoustic-gravity waves in the ocean to detect tsunamis [3], as the sound travels much faster in water than the tsunami wave itself. To model the acoustic-gravity waves, we consider the Navier-Stokes equations for an iviscid, weakly compressible and free-surface fluid and aim at a linear approximation of these nonlinear equations that retains the two types of waves.

The ocean is supposed to be close to an equilibrium, stratified, with a varying density $\rho$ and temperature $T$. Its velocity is denoted $\mathbf{U}$, its pressure, $p$. The Navier-Stokes equations in Lagrangian coordinates are linearized around a steady state for the ocean at rest : there is no mean current and the pressure, density and temperature have only vertical variations. This corresponds to an asymptotic expansion with $\epsilon \ll 1$ :

$$\mathbf{U} = \epsilon \mathbf{U}_1 + \mathcal{O}(\epsilon^2), \quad \rho = \rho_0 + \epsilon \rho_1 + \mathcal{O}(\epsilon^2), \quad p = p_0 + \epsilon p_1 + \mathcal{O}(\epsilon^2).$$

From the linearized equations a wave equation for the velocity is derived,

$$\rho_0 \frac{\partial^2 \mathbf{U}_1}{\partial t^2} - \nabla_\xi \left( \rho_0 c_0^2 \nabla_\xi \cdot \mathbf{U}_1 \right) - \left( \nabla_\xi \mathbf{U}_1 \right)^T \rho_0 \mathbf{g} + \rho_0 \nabla_\xi \cdot \mathbf{U}_1 \ \mathbf{g} = 0. \tag{1}$$

Where $c$ is the speed of sound in water. By writing the variational formulation of (1) with its boundary conditions, we obtain a model with a bilinear form. The form is symmetric and positive, which ensures the well-posedness of the model.

Two aspects of this wave equation are then studied. First, an asymptotic limit for small Mach number is carried out and we show that our limit model is a generalization of known models for incompressible flow with varying density ([4]). Second, the system is put back to Eulerian coordinates and compared to other models in the literature. We show that the equations inside the domain are the same as in [2]. Our model retains also more complexity than the well-known wave equation for the fluid potential, which is obtained from more approximations and is widely used for practical applications ([4], [1]).

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