Nonlinear Domain Decomposition and Multilevel methods for solving Phase-Field Fracture Problems

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The phase-field approach for modeling crack propagation is one of the state-of-the-art strategies in the field of fracture mechanics. The phase-field method gives rise to strongly nonlinear coupled partial differential equations, as one has to solve for the displacement field and an additional damaged variable that characterizes the material state from intact to fully broken. The modeling of such problems is computationally challenging due to the non-convex nature of the underlying energy functional and the other difficulties in the solution scheme arises due to the ill-conditioning caused by highly varying damage variable.
We propose to solve the arising nonlinear problems efficiently using multilevel [2] and domain decomposition [1] methods. The proposed multilevel method employs a novel level-dependent energy functional that combines a fine-level description of the crack paths with the coarse level discretization. In particular, we discuss different techniques to construct a hierarchy of suitable subspaces, related to different levels or subdomains. For the domain decomposition approach, we take advantage of the field-split method to decouple the problem into two sets that are related to the displacement and the damage variable. This spit conveniently gives rise to the additive and multiplicative variants of the Schwarz preconditioned inexact Newton method to solve the nonlinear system.
In this talk, we will demonstrate the convergence behavior of the proposed solution strategies using several benchmark problems. We will also compare these methods with the widely-used alternate minimization method and show the benefits of our solution schemes. Finally, we will also illustrate the strong and weak scaling properties of the overall simulation framework [3].

[1] A. Kopaničáková, H. Kothari, R. Krause. *Nonlinear Field-split Preconditioners for Solving Monolithic Phase-field Models of Brittle Fracture.* Under review in Computer Methods in Applied Mechanics and Engineering, 2022.

[2] A. Kopaničáková, R. Krause. *Recursive multilevel trust region method with application to fully monolithic phase-field models of brittle fracture.* Computer Methods in Applied Mechanics and Engineering, **360**, 112720, 2020.

[3] P. Zulian, A. Kopaničáková, M. G. C. Nestola, N. Fadel, A. Fink, J. VandeVondele, R. Krause. *Large scale simulation of pressure induced phase-field fracture propagation using Utopia.* CCF Transactions on High Performance Computing, 2021. doi :10.1007/s42514-021-00069-6.

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