Convergence analysis of ParaOpt algorithm for unstables systems

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Parallel solving of optimality systems arising in partial differential equations (PDE) constrained optimization can be performed by applying suitable time parallel algorithm to solve the forward and backward evolution problems derived from the optimality system. In this talk, we consider a recent time parallel algorithm, namely the ParaOpt algorithm, which is a two-level parareal method for the coupled forward and backward equations. We present a convergence analysis of this algorithm in the case of unstable systems. More precisely, we consider the optimal control problem associated with the cost functional

$$J(\nu) = \frac{1}{2}\|y(T) - y_{target}\|^2 + \frac{\alpha}{2}\int_0^T \|\nu(t)\|^2,$$

where α is a fixed regularization parameter, y_{target} is a target state, and the evolution of the state function $y : [0,T] \longrightarrow \mathbb{R}$ is described by the linear equation

$$\begin{aligned} \dot{y}(t) =& \sigma y(t) + \nu(t), t \in [0,T] \\ y(0) =& y_0 \end{aligned}$$

with $\sigma > 0$. We will then focus on the spectral radius ρ of the iteration matrix and obtain the following result.

Let $L \in \mathbb{N}$ and $\Delta T = T/L$. According to ParaOpt idea, we consider a subdivision $\cup_{\ell=0}^{L-1}[T_\ell, T_{\ell+1}]$ with $T_\ell = \ell\Delta T$, and $\Delta t, \delta t$ a coast and fine time step respectively such that $\delta t \leq \Delta t \leq \Delta T$. In time discretization formulation, we define

$$\begin{aligned} \beta &= (1+\sigma\Delta t)^{\frac{\Delta T}{\Delta t}} & \beta_{\delta t} &= \beta = (1+\sigma\delta t)^{\frac{\Delta T}{\delta t}} & \delta\beta &= \beta - \beta_{\delta t} \\ \gamma &= \frac{\beta^2-1}{\sigma(2+\sigma\Delta t)} & \gamma_{\delta t} &= \frac{\beta_{\delta t}^2-1}{\sigma(2+\sigma\delta t)} & \delta\gamma &= \gamma - \gamma_{\delta t}, \\ \tau &= \beta - \gamma\frac{\delta\beta}{\delta\gamma}. \end{aligned}$$

We consider the mapping $R(x) = x^{2L-1} - x^{2L-2} - x + \frac{1}{\beta}$, τ_0 such that $R(\tau_0) \geq 0$ and

$$L_0 = \frac{\beta - \tau}{\gamma(\tau - \tau_0)}.$$

Keeping those notations we get :

Theorem 1. *Let $\sigma > 0, \alpha, T, L, \Delta t, \delta t$ be fixed such that the $R(\tau) > 0$. If $L > \alpha L_0$, then the spectral radius of iteration matrix satisfies*

$$\rho < \sigma(\Delta t - \delta t)\left[\frac{1}{2} + \left(\frac{\sigma}{2}(\Delta t - \delta t) + 1\right)e^{2\sigma\Delta T}\right].$$

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