Surface tension effects between two immiscible Stokes flows simulated using unfitted hybrid high order methods

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We consider the equilibrium of two immiscible, incompressible Stokes fluids with surface tension effects at their interface [3]. The interface splits the computational domain into two subdomains, and each subdomain is occupied by a fluid governed by the steady, incompressible Stokes equations. At the interface, the fluid velocities are continuous, whereas the jump of the normal total stress is proportional to the curvature of the interface (Laplace's law). These equations define the so-called *Stokes interface problem*. Instead, in the so-called *interface equilibrium problem*, the Stokes interface problem has to be completed by requiring that the normal velocity at the interface is zero. This last condition enforces the equilibrium and essentially prescribes the shape of the interface.
To solve the Stokes interface problem, we consider an unfitted hybrid high-order (HHO) method. In a nutshell, the method is called unfitted because it employs a mesh that does not fit the interface, that is, the interface can cut arbitrarily through some of the mesh cells. The use of unfitted meshes greatly simplifies the meshing process since such meshes can be chosen in a very simple manner. The jump conditions at the interface are then enforced by means of a consistent penalty technique inspired by Nitsche's method to weakly enforce non-homogeneous Dirichlet conditions. The HHO method employs hybrid unknowns (face- and cell-based) for the velocity and only cell-based unknowns for the pressure. Among its assets, when applied to incompressible Stokes flows, the HHO method is inf-sup stable, locally conservative, and supports polytopal meshes. Moreover, the HHO method is computationally efficient, owing to its compact stencil and to the possibility of a local elimination of the cell velocity unknowns by a static condensation procedure. Unfitted HHO methods for elliptic interface problems have been derived in [1], where a local cell-agglomeration procedure is used to counter the adverse effects of unfavorably cut cells. Unfitted HHO methods for the Stokes interface problem have been analyzed in [2], which constitutes the starting point of the present work.
We consider the interface equilibrium problem with no body forces and a shear flow prescribed at infinity. Then the equilibrium interface is an ellipse (the area of the ellipse still remains a free parameter in the problem), and, being the fluid viscosities fixed, the ellipse eccentricity depends upon the ratio of the prescribed shear to surface tension (also called capillary number). When the capillary number is zero (no prescribed shear), the ellipse eccentricity is zero, that is, the ellipse becomes a circle. More generally, we investigate numerically the dependence of the equilibrium ellipse on the capillary number and find a linear relationship. This result confirms the theoretical analysis from [3].

[1] E. Burman, M. Cicuttin, G. Delay, A. Ern. *An unfitted hybrid high-order method with cell agglomeration for elliptic interface problems.* SIAM J. Sci. Comput., **43(2)**, A859–A882, 2021. doi :10.1137/19M1285901.

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[3] G. I. Taylor. *The formation of emulsions in definable fields of flow.* Proc. Roy. Soc. London Ser. A, **146(858)**, 501–523, 1934.

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