On implicit-explicit well-balanced Lagrange-projection schemes for two-layer shallow water equations

Alessia DEL GROSSO, LMV - Versailles, France
Manuel J. CASTRO DIAZ, Universidad de Málaga - Málaga, Spain
Christophe CHALONS, LMV - Versailles, France
Tomás MORALES DE LUNA, Universidad de Córdoba - Córdoba, Spain

This presentation concerns well-balanced Lagrange-projection schemes applied to the two-layer shallow water system. With such a model, we assume the fluid to be composed of two superimposed layers of immiscible liquids where the upper one has a smaller density [3]. In particular, we are interested in situations in which the ratio of the two densities is close to one, which often happen in geophysical flows. However, in such cases, the system is only conditionally hyperbolic as complex internal eigenvalues could appear [1]. A further difficulty is related to the presence of two velocities, one for each layer, due to which it is not straightforward to understand how to apply the Lagrange-projection strategy to this system. One idea could be to implement the Lagrange-projection approach for each layer and then to couple them. However, it is known that this kind of decoupled strategy usually leads to the presence of spurious oscillations in the numerical results [1]. Hence, here we consider a different coupled approach and, in particular, we propose a formulation of the mathematical model in Lagrangian coordinates. We also consider a different interpretation of the Lagrange-projection approach, namely the acoustic-transport splitting [2]. The latter implies the decomposition of the different phenomena of the model, resulting in two different systems, the acoustic and transport ones. For the former, we design an approximate Riemann solver based on a relaxation approach and then use the associated Godunov-type scheme. We also underline that the resulting approximation can be exploited for the Lagrangian system. Finally, let us recall that the acoustic-transport splitting (or the Lagrange-projection decomposition) can be particularly interesting in subsonic regimes, where the acoustic waves are much faster than the transport ones. This means that an implicit approximation of the acoustic system could lead to the design of very fast numerical schemes as we would neglect the acoustic CFL condition on the time step. For this reason, we propose both an explicit and an implicit strategy for the acoustic equations, while keeping an explicit approximation for the transport step.

[1] M. Castro, E. Fernández-Nieto, J. G. Vida, C. Madroñal. *Numerical treatment of the loss of hyperbolicity of the two-layer shallow-water system.* Journal of Scientific Computing, **48**, 2011.

[2] C. Chalons, P. Kestener, S. Kokh, M. Stauffert. *A large time-step and well-balanced lagrange-projection type scheme for the shallow-water equations.* Communications in Mathematical Sciences, **15**, 2016.

[3] E. Fernández-Nieto, M. Castro, C. Madroñal. *On an intermediate field capturing riemann solver based on a parabolic viscosity matrix for the two-layer shallow water system.* Journal of Scientific Computing, **48**, 2011.

Contact : alessia.del-grosso@ens.uvsq.fr