Numerical tools and examples in optimal control

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An Optimal Control Problem (OCP) is an infinite dimensional optimization problem with algebraic and differential constraints. A simple example of OCP in Lagrange form is the following. Find a command law which steers a dynamical control system from an initial configuration to a target point while minimizing an objective function of integral form. The optimal trajectory can be found as the projection of an extremal, solution of necessary conditions given by the Pontrjagin Maximum Principle (PMP). Generally, computing an extremal amounts to solve a set of nonlinear equations given by the PMP, the so-called shooting equations.

Numerical methods to compute approximation of local solutions of OCP's fall into two main categories. Indirect methods based on Pontrjagin principle are fast and accurate but require more work, in terms of mathematical analysis and a priori knowledge on the structure of the solution. They are also sensitive to the initial guess due to the underlying Newton method. On the contrary, direct transcription approaches offer a good trade-off bewteen robustness, accuracy and modelling efforts. For challenging problems, one may start with a direct method to find a rough solution and then refine it through an indirect method. An alternative approach is to replace the direct part by a homotopy process.

Differential homotopy techniques consist in computing a path of solutions of a one-parameter family of nonlinear equations. Under suitable assumptions, a path of zeros is a differential curve which can be computed by Predictor-Corrector (PC) algorithms. The prediction part consists in an integration step and can be done by Runge-Kutta schemes. The correction is usually done by a simplified Newton method and guarantee a better approximation of the path of zeros. The key point of homotopy methods is the step-length adaption to get, efficiently, a good approximation of the curve.

In the context of optimal control, homotopy methods can be used in many ways. We can use globalization techniques to overcome the sensitivity of the shooting method. We can apply penalizations to handle inequality state constraints but also regularizations to simplify the computation of control laws. Regularizations on the dynamics can also be considered. These applications of homotopy techniques are quite usual. More original uses may be found in two-dimensional geometry where differential homotopy can be applied to compute the so-called wavefronts, conjugate and cut loci. Another possibility is to use homotopy to classify extremals with respect to two parameters in order to compute for instance an optimal synthesis. Finally, in the regular case, when the strict Legendre-Clebsh condition is satisfied, then a PC algorithm can be used to solve the DAE given by the PMP to compute numerically the extremal together with the optimal control law.

In this talk, I will report ongoing work on a Julia package, extending previous developments in numerical methods for optimal control including `hampath` (`http://hampath.org`) and `ct: control toolbox` (`https://ct.gitlabpages.inria.fr/gallery/`). The focus is on indirect methods and path following combined with automatic differentiation.

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