Non-intrusive implementation of multiscale finite element methods

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The multiscale finite element method (MsFEM) is a finite element (FE) approach that allows to solve partial differential equations (PDEs) with highly oscillatory coefficients on a coarse mesh, i.e., a mesh with elements of size much larger than the characteristic scale of the oscillations [3, 2]. To do so, MsFEMs use pre-computed basis functions that solve *local* problems mimicking the problem to be solved on a domain of the size of the coarse mesh elements. The basis functions are then coupled through a Galerkin approximation of the *global* problem.

Standard FEM codes used in industry are based on problem-independent (e.g. polynomial) basis functions such that the resolution of the Galerkin approximation can be automated for a large variety of problems. The MsFEM is, on the contrary, particularly interesting when the same problem is to be solved repeatedly for varying inputs such as boundary conditions and right-hand side terms. However, the problem-dependent nature of the MsFEM approach seems to require that large parts of any existing FEM code either be adapted or developed from scratch. This makes its use very intrusive and hinders the adoptation of MsFEMs in industry. Seeking to facilitate the implementation of MsFEMs in industrial, non-academic codes, we have investigated how MsFEMs can be adapted to benefit from an existing FEM code as an integral part of the approach [1] (possibly at the cost of a marginal loss in their efficiency).

In this contribution, we will show on the example of a multiscale diffusion problem how the MsFEM can be reformulated as an effective problem with a diffusion coefficient that is piecewise constant on the coarse mesh. Since the small scale of the oscillations has vanished in the effective problem, it can be solved by a standard FEM code on the coarse mesh. This can directly be translated to a non-intrusive MsFEM that can fully exploit any FEM code currently in use. We will then generalize this non-intrusive MsFEM approach to different MsFEM variants and to more complicated PDEs.

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