Local flux reconstruction for diffusion problems with discontinuous coefficients

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This work is part of the PhD thesis of A. Gouasmi. One of the goals of the thesis is to reconstruct locally conservative fluxes for finite element approximations of different boundary/interface model problems. Conservative fluxes are of interest for physicists in many applications such as heat transfer or porous media problems, but they are also used to define simple a posteriori error estimators which are employed in adaptive mesh refinement. Thus, another goal of this work is to carry on a posteriori error analysis based on the numerical fluxes.

The literature on flux reconstruction is very rich ; in this work we follow the approach proposed in [1] for the Poisson problem in a polygonal domain, discretized by standard finite element methods (conforming, nonconforming and discontinuous Galerkin). The first part of the talk is devoted to the extension of the previous paper to the case of a diffusion problem with highly discontinuous coefficients. We only consider a conforming finite element approximation, which is the most difficult to treat, and we focus on the robustness of the reconstruction and of the a posteriori error analysis with respect to both the mesh and the coefficients.

In the second part of the talk, we consider an elliptic problem with an interface which does not follow the mesh, and with diffusion coefficients which are discontinuous across the interface. For the numerical treatment of the interface, we employ the recent CutFEM method, which is based on Nitsche's extended finite element method cf. [6] with additional stabilization terms ([2], [3]) in order to improve its robustness with respect to the mesh-interface geometry. Although CutFEM is now used in many applications, there are few works on the a posteriori error analysis for this method. One can cite [4] for residual-based a posteriori error estimators and [5] for locally reconstructed fluxes, both for a Poisson model problem but where the boundary is not aligned with the mesh.

In this talk, we will present our first results concerning the extension to a diffusion problem in the presence of an interface not aligned with the mesh.

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