Multiscale Finite Element method for incompressible flow

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Multi-scale problems arise in numerous engineering fields such as reservoir engineering, flows through fractured porous media, flows in nuclear reactor cores, etc. In these media with many obstacles of various sizes, the macroscopic flow is directly influenced by local phenomena occurring at the finest scales. Thus, these problems require a very fine mesh to resolve all the details. Despite the continuous increase in computer resources, these are insufficient to perform classical finite element simulations with an accuracy allowing correct resolution of the finest scales of the flow. To overcome this limitation, various multi-scale methods have been developed to attempt to resolve scales below the coarse mesh scale by incorporating local computations into a global problem which is defined only on a coarse mesh. In this project, we focus on the multi-scale finite element method.

A multi-scale finite element method uses a coarse mesh on which one defines basis functions which are no longer the classical polynomial basis functions of finite elements, but which solve fluid mechanics equations on the elements of the coarse mesh. These functions are themselves numerically approximated on a fine mesh considering all the geometric details, which gives the multi-scale aspect of this method.

Based on the work of [1, 2], we propose to develop an enriched multi-scale finite element method (MsFEM) in the vein of the classical non-conforming Crouzeix-Raviart finite element method with high-order weighting functions. In this project, we proposed to analyse the convergence of the multi-scale method, including errors due to the discretization on the coarse mesh, and those due to the approximation of the basis functions on the fine meshes. We will implement the multi-scale finite element methods developed in a high-performance computing framework in FreeFEM : the basis functions being independent of each other, their approximations as well as the assembly of the macroscopic problem can be carried out in parallel. One of the objectives of this work is to carry out simulations of turbulent flows at high Reynolds number in a highly congested environment.

[1] Q. Feng. *Development of a multiscale finite element method for incompressible flows in heterogeneous media.* phdthesis, Université Paris Saclay (COmUE), 2019.

[2] G. Jankowiak, A. Lozinski. *Non-Conforming Multiscale Finite Element Method for Stokes Flows in Heterogeneous Media. Part II : error estimates for periodic microstructure.* arXiv :1802.04389 [math], 2018. ArXiv : 1802.04389.

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