# A Posteriori error estimator for a coupled Stokes problem

**Hussein ALBAZZAL**, Laboratoire de Mathématiques de Besançon - Besançon
**Alexei LOZINSKI**, Laboratoire de Mathématiques de Besançon - Besançon
**Roberta TITTARELLI**, FEMTO-ST - Besançon

This work deals with an efficient numerical solution of the Stokes problem in a 2D domain coming from the serpentine channels of a proton-exchange membrane fuel cell [2] as showed in Fig. 1. In order to speed up the resolution, a 1D model in the part of the channel that is rectangular ($\Omega_{1D}$ in Fig.1) and a P2-P1 Finite Element resolution in the corners ($\Omega_{2D}$ in Fig.1), in a similar spirit of [3], are proposed. The 1D model consists in an analytical solution under suitable assumptions and the final coupled model on the whole domain is formulated with reasonable interface conditions. To find a good compromise between precision and computational efficiency, the challenge is double : how to choose a suitable position of the interface between the 1D and the 2D models and how to control the discretisation error in the corners.

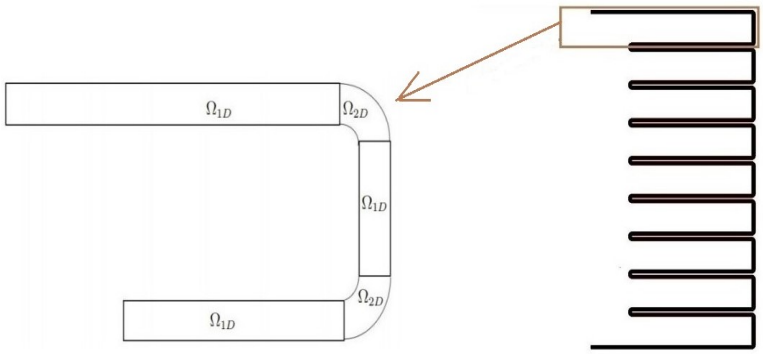


FIGURE 1 – On the left, an example of the decomposition of the domain of the coupled 1D-2D Stokes model. On the right, an example of a whole serpentine channel of a fuel cell.

A test to choose a good position for the interface and an a *posteriori* equilibrated error estimator, inspired by [1], where the FE method is applied are proposed and their relation are discussed. The upper and lower bounds of the energy error on the whole domain by the estimator are proved. Some numerical results are showed to validate the upper bound and to provide an algorithm to couple efficiently these 1D-2D models. Since the inf-sup constant of the Stokes problem is unknown and is involved in the error estimator, an open subject is to find an estimation of it.

[1] A. Hannukainen, R. Stenberg, M. Vohralík. *A unified framework for a posteriori error estimation for the stokes problem.* Numerische Mathematik, **122(4)**, 725–769, 2012.

[2] S. Karvonen, T. Hottinen, J. Saarinen, O. Himanen. *Modeling of flow field in polymer electrolyte membrane fuel cell.* Journal of power sources, **161(2)**, 876–884, 2006.

[3] M. Tayachi, A. Rousseau, E. Blayo, N. Goutal, V. Martin. *Design and analysis of a schwarz coupling method for a dimensionally heterogeneous problem.* International Journal for Numerical Methods in Fluids, **75(6)**, 446–465, 2014.

Contact : `hussein.albazzal@univ-fcomte.fr`