

Lebesgue Induction Principle and Tonelli's Theorem in Coq

Sylvie BOLDO, Inria, LMF - 91190, Gif sur-Yvette, France.

François CLÉMENT, Inria - 2 rue Simone Iff, 75589 Paris, France. **Micaela MAYERO**, LIPN, Université Paris 13 - Villetaneuse, France. **Houda MOUHCINE**, Inria, LMF - 91190, Gif sur-Yvette, France.

Résumé

Lebesgue integration is a well-known mathematical tool, used for instance in probability theory, real analysis, and numerical mathematics. Thus, its formalization in a proof assistant is to be designed to fit different goals and projects. Once Lebesgue integral is formally defined and the first lemmas are proved, the question of the convenience of the formalization naturally arises. To check it, a useful extension is the Tonelli theorem, stating that the (double) integral of a nonnegative measurable function of two variables can be computed by iterated integrals, and allowing to switch the order of integration. This article describes the formal definition and proof in Coq of product σ -algebras, product measures and their uniqueness, the construction of iterated integrals, up to the Tonelli theorem. We also advertise the Lebesgue induction principle provided by an inductive type for nonnegative measurable functions.