# Non-overlapping Domain Decomposition Methods for Elliptic Control Problems

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**Résumé**

In this talk, we will present some non-overlapping Domain Decomposition Methods (DDM) applied to the optimal control problems arising from elliptic partial differential equations (PDE). This problem reads as : for a given state $y$ governed by a stationary heat conduction equation, we wish to drive the solution of this PDE to a desired state $\hat{y}$ through a control $u$. The goal is to find the optimal control $u^*$ which minimizes the discrepancy between these states (i.e. original state $y$ and desired state $\hat{y}$). We first use the Lagrange multiplier approach to derive a forward-backward system [1]. Instead of the commonly used $L^2$ regularization, we show that this forward-backward system can be simplified into one single second order PDE related to the state $y$ by applying an $H^{-1}$ regularization [2, 3]. This avoids solving a coupled BiLaplacian problem. The simplified problem can then be solved with DDM. We provide the convergence analysis for Dirichlet-Neumann and Neumann-Neumann methods along with some numerical results.

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[2] U. Langer, O. Steinbach, F. Tröltzsch, H. Yang. *Space-time finite element discretization of parabolic optimal control problems with energy regularization.* SIAM Journal on Numerical Analysis, **59(2)**, 675–695, 2021. doi :10.1137/20M1332980.

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