Numerical simulations and statistical description of the quantum turbulence modelled by HVBK equations

Zhentong ZHANG, URN, Laboratoire de Mathématiques Raphaël Salem - Rouen
Ionut DANAILA, URN, Laboratoire de Mathématiques Raphaël Salem - Rouen
Luminita DANAILA, URN, Laboratoire de Morphodynamique Continentale et Côtière - Rouen

The liquid helium below the lambda transition temperature (also called Helium II) presents many intriguing properties [3]. The present study focuses on the effect of the mutual friction to the scale-by-scale energy budget (or, energy cascade) in the quantum turbulence by using direct numerical simulation (DNS) of the two-fluid HVBK (Hall-Vinen-Bekarevich-Kalat) model, which is a self-consistent models to study the superfluid turbulence at macroscopic scales [4, 5]. We solved numerically the system of incompressible HVBK equations using Fourier pseudo-spectral methods. Direct numerical simulations were performed by adapting a Navier-Stokes code that proved efficient and accurate in computing high-order statistics of turbulent flows [2]. Periodic boundary conditions are applied to a computational box of length 2π. Grid resolution was 512^3, which was sufficient to reach a moderate $Re_\lambda = 100$, based on Taylor's microscale. To achieve a quasi-stationary homogeneous isotropic turbulence, an additional forcing term was added in the momentum equations of at large scales. To statistically describe the scale-by-scale energy exchanges, we use the same method as in [1] to derive transport equations for the second-order structure functions from the incompressible HVBK equations, which account for the contribution of the mutual friction term. By analyse the numerical results we found that for high temperatures ($\rho^n/\rho^s = 10$), the normal fluid dominates. The dynamics is similar to the classical turbulence. A classical '4/3-rds law' emerges in the inertial range. The mutual friction term and the inertial term have opposed signs in the superfluid. They nearly balance each other for all scales, thus suggesting the mutual friction term acts as a source sinking, with the same scaling as the inertial term. For low temperatures ($\rho^n/\rho^s = 0.1$), the superfluid, dominant over the normal fluid, is unable to efficiently dissipate the injected energy. Therefore, the energy must go the traditional way through the energy cascade and eventually be dissipated in the superfluid by the artificial viscosity introduced in the model, which makes the superfluid turbulence behaves nearly as classical turbulence. In summary, the mutual friction force represents a bridge for the energy exchange between the two fluids, it does not modify the scaling law of the inertial term. Instead, it follows the scaling law of the inertial term. As a consequence, the HVBK model offers a reliable framework for studying superfluid turbulence, with a key account of the energy transfer rate through the mutual friction.

[1] L. Danaila, L. Voivenel, E. Varea. *Self similarity criteria in anisotropic flows with viscosity stratification.* Phys. of Fluids, **29**, 020705, 2017.

[2] M. Gauding, L. Danaila, E. Varea. *High-order structure functions for passive scalar fed by a mean gradient.* International Journal of Heat and Fluid Flow, **67**, 86–93, 2017.

[3] M. Kobayashi, P. Parnaudeau, F. Luddens, C. Lothodé, L. Danaila, M. Brachet, I. Danaila. *Quantum turbulence simulations using the Gross-Pitaevskii equation : High-performance computing and new numerical benchmarks.* Computer Physics Communications, **258**, 107579, 2021.

[4] P.-E. Roche, C. Barenghi, E. Lévêque. *Quantum turbulence at finite temperature : the two-fluids cascade.* European Phys. Lett., **87**, 54006, 2009.

[5] J. Salort, B. Chabaud, E. Lévêque, P.-E. Roche. *Energy cascade and the four-fifths law in superfluid turbulence.* European Phys. Lett., **97**, 34006, 2012.

Contact : `ionut.danaila@univ-rouen.fr`