Anisotropic nonlinear elliptic equations

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The present talk is devoted to the existence of two nontrivial solutions for the following nonlinear elliptic equations driven by an anisotropic Laplacian operator

$$\begin{cases} -\Delta_{\vec{p}} u = \lambda f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{$D_\lambda^{\vec{p}}$}$$

where $\Delta_{\vec{p}} u = \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(\left|\frac{\partial u}{\partial x_i}\right|^{p_i-2}\frac{\partial u}{\partial x_i}\right)$ is the anisotropic $p-$Laplacian operator, $\lambda \in]0,+\infty[$ and $f : \Omega \times R \to R$ is an L^1-Carathéodory function.
Our main tool is a two critical points theorem established in [3]. Such critical point result is an appropriate combination of the local minimum theorem obtained in [2], with the classical and seminal Ambrosetti-Rabinowitz theorem (see [1]). The functional framework involves the anisotropic Sobolev spaces.
The following Theorem is a special case of our main result.

Théorème 1. *[4] There is $\eta^* > 0$ such that, for each $\eta \in]0,\eta^*[$, the problem*

$$\begin{cases} -\Delta_{\vec{p}} u = \eta u^{(p^- -2)} + u^{p^+} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

has at least two positive weak solutions.

Références

[1] A. Ambrosetti, P.H. Rabinowitz, *Dual variational methods in critical point theory and applications*, J. Funct. Anal., **14** (1973) 349–381.

[2] G. Bonanno, *A critical point theorem via the Ekeland variational principle*, Nonlinear Anal., **75** (2012) 2992–3007.

[3] G. Bonanno, G. D'Aguì, *Two non-zero solutions for elliptic Dirichlet problems*, Z. Anal. Anwendungen, **35** (2016), 449–464.

[4] G. Bonanno, G. D'Aguì, A. Sciammetta, *Existence of two positive solutions for anisotropic nonlinear elliptic equations*, Advances in Differential Equations, vol. **26** (2021), 229-258.

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