A posteriori error estimators for a non-symmetric eigenvalue problem. Application to a Boltzmann operator and a reduced basis method in neutronics.

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We are interested in a parametrized two-group neutron diffusion equations [3]. Let us define the so-called *high-fidelity* generalized eigenvalue problem generated from a finite-element discretization by

$$\begin{aligned} &\text{Find } (u_{\mu,h}, \lambda_{\mu,h}) \in \mathbb{R}^{N_h} \times \mathbb{R} \text{ such that} \\ &A_{\mu,h} u_{\mu,h} = \lambda_{\mu,h} B_{\mu,h} u_{\mu,h}, \end{aligned} \tag{1}$$

where $A_{\mu,h} \in \mathbb{R}^{N_h \times N_h}$ is assumed invertible, $B_{\mu,h} \in \mathbb{R}^{N_h \times N_h}$ is assumed nonnegative, N_h denotes the dimension of the approximation space, and the parameter $\mu \in \mathcal{P}$ stands for a given configuration of the core, also called loading pattern. Let us assume the existence of an eigenvalue of smallest magnitude that is considered unique [1]. In some applications such as the loading plan optimization of a nuclear reactor [5], exploring the manifold $\mathcal{M}^{\mathrm{HF}} = \{(u_{\mu,h}, \lambda_{\mu,h}),\ \mu \in \mathcal{P}\}$ is prohibitively expensive in terms of computational cost.
Our approach is to build a reduced basis for the manifold $\mathcal{M}^{\mathrm{RB}} = \{(u_{\mu,N}, \lambda_{\mu,N}),\ \mu \in \mathcal{P}\}$.
The associated reduced problem writes

$$\begin{aligned} &\text{Find } (c_{\mu,N}, \lambda_{\mu,N}) \in \mathbb{R}^{N} \times \mathbb{R} \text{ such that} \\ &A_{\mu,N} c_{\mu,N} = \lambda_{\mu,N} B_{\mu,N} c_{\mu,N} \quad \text{and} \quad u_{\mu,N} = V_N c_{\mu,N}, \end{aligned} \tag{2}$$

The reduced basis method is made up of two stages. In an *offline* stage, we first build a reduced space V_N with a Greedy procedure, from a few computations of the high-fidelity solver. This procedure is iterative and is based on an a posteriori error estimator such that the variations over μ of the latter are similar to those of the former. We call this quantity an posteriori error estimator [2, 4]. Then, an *online* stage, for a given core configuration μ, solving the reduced problem gives an approximation of the solution within a much smaller computational time as the one performed by the high-fidelity solver.

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