Optimisation of Robin coefficients from the optimal control point of view

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The goal of this talk is to present recent qualitative results related to the optimisation of Robin coefficients in the optimal control of elliptic equations. Broadly speaking, our goal is to present insights into the following class of problems : considering an elliptic equation

$$\begin{cases} -\Delta u_\beta = F(x, u_\beta) & \text{in } \Omega \\ \partial_\nu u_\beta + \beta(x) u_\beta = 0 & \text{on } \partial\Omega, \end{cases}$$

what can be said about the solutions of

$$\max_{\beta \in B(\partial\Omega)} / \min J(u_\beta)?$$

In the equation and the optimisation problem above the Robin coefficient β belongs to the admissible class

$$B(\partial\Omega) := \left\{ \beta \in L^\infty(\partial\Omega) : 0 \leq \beta \leq 1 \text{ a.e.}, \int_{\partial\Omega} \beta = V_0 \right\}$$

where V_0 is a fixed volume constraint. We shall present results related to two classes of functionals J :

1. First, we consider the case of energetic functionals (in other words, we try to minimise the natural energy associated with the PDE). Here, we can get an almost explicit description of optimisers.
2. Second, following recent advances in the qualitative analysis of bilinear optimal control problems, we propose an in-depth analysis of non-energetic functionals. These functionals typically write
$$J(u_\beta) = \int_\Omega j_1(u_\beta) + \int_{\partial\Omega} j_2(u_\beta).$$
A crucial information to derive on this class of functionals is the so-called *bang-bang property* : are optimisers extreme points of the admissible set $B(\partial\Omega)$? We prove that this is the case under mild monotonicity assumptions.

This talk is based on a joint work [1] with Y. Privat.

[1] I. Mazari, Y. Privat. *Qualitative analysis of optimisation problems with respect to non-constant robin coefficients.* Submitted, 2021.

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